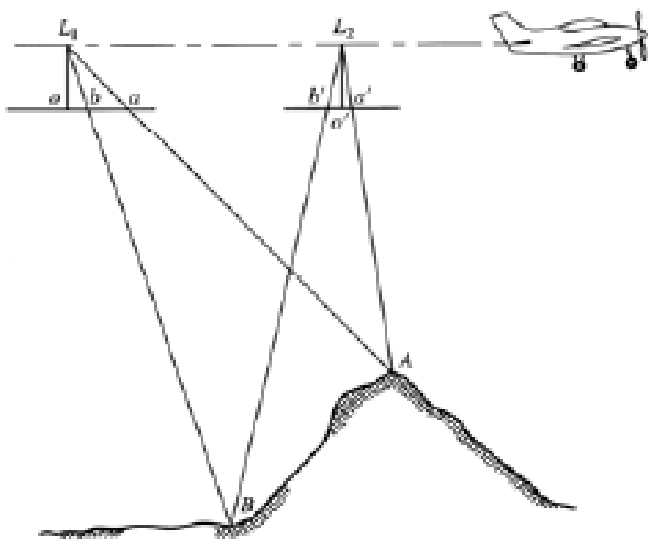
# STEREOSCOPIC PARALLAX

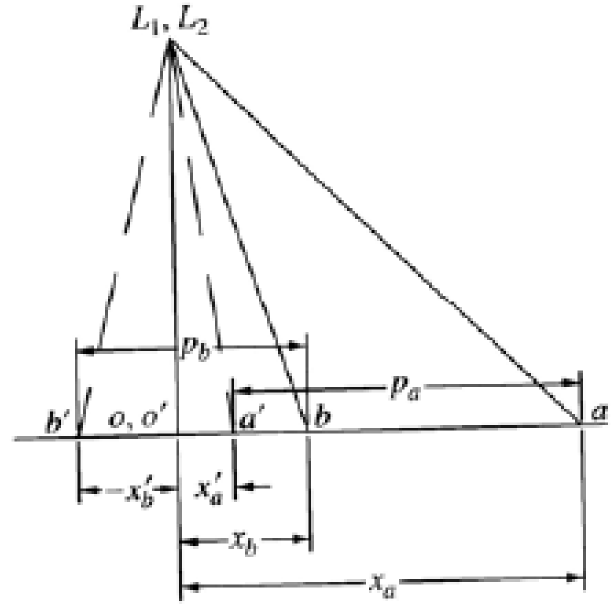
# PARALLAX

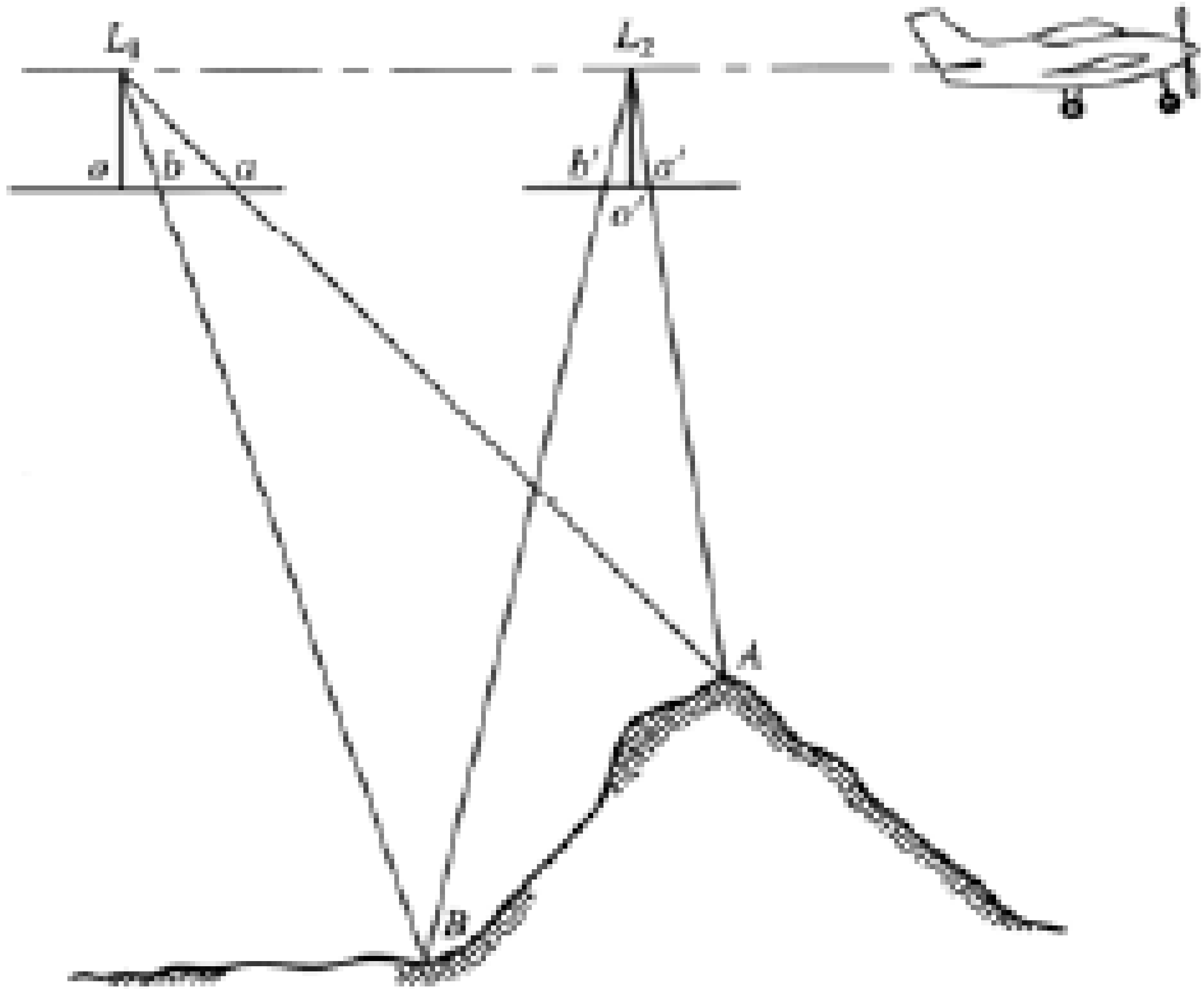
- Apparent shift in the position of an object, with respect to a frame of reference, caused by a shift in the position of observation
- Change in position of an image from one photo to the next is caused by aircraft's motion
  - Called stereoscopic parallax, x parallax, or simply parallax
- Two important aspects of stereoscopic parallax
  - Parallax of any point is directly related to the elevation of the point
  - Parallax is greater for high points than for low points

# STEREOSCOPIC PARALLAX

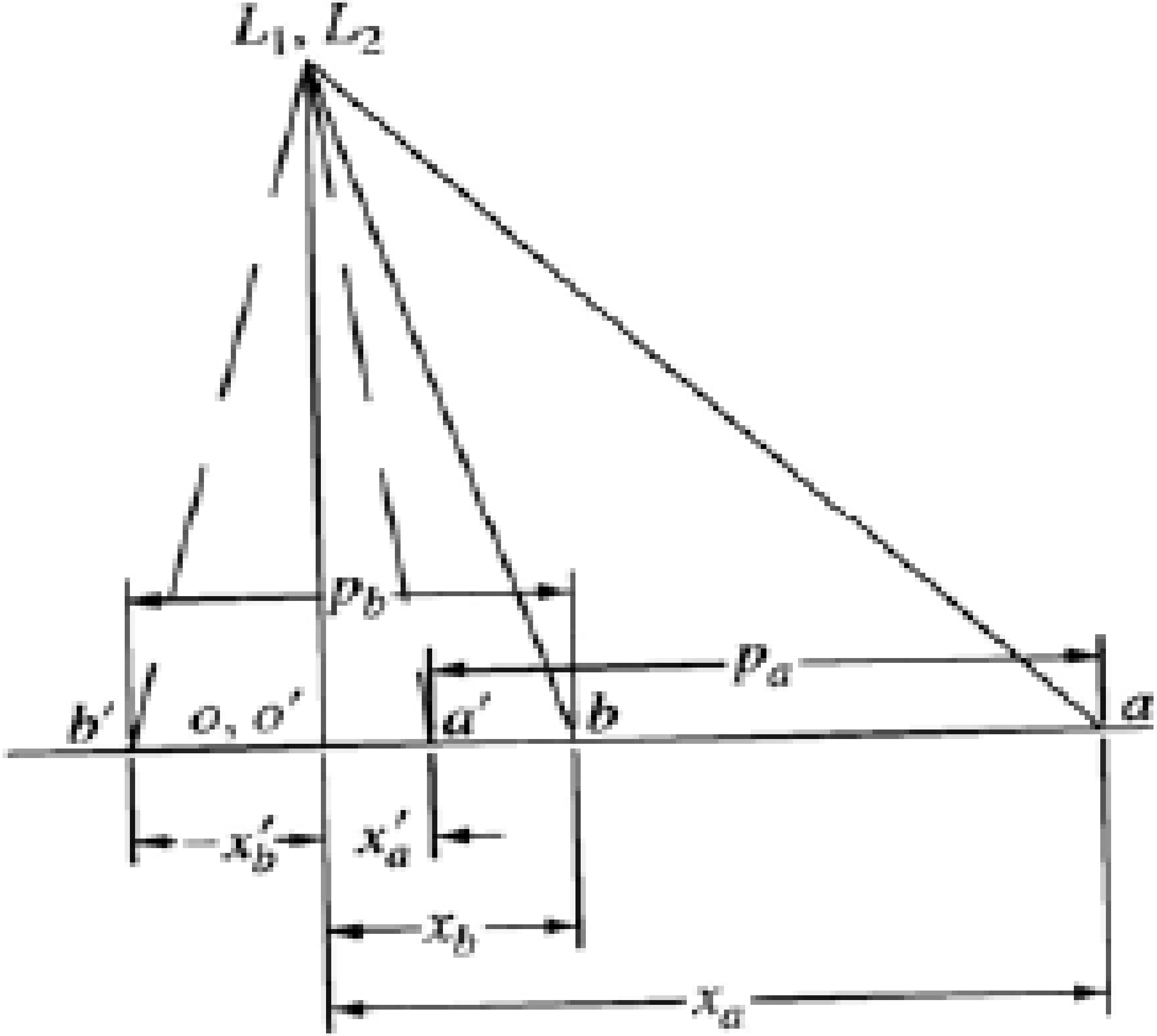
$$P_a = X_a - X'_a$$





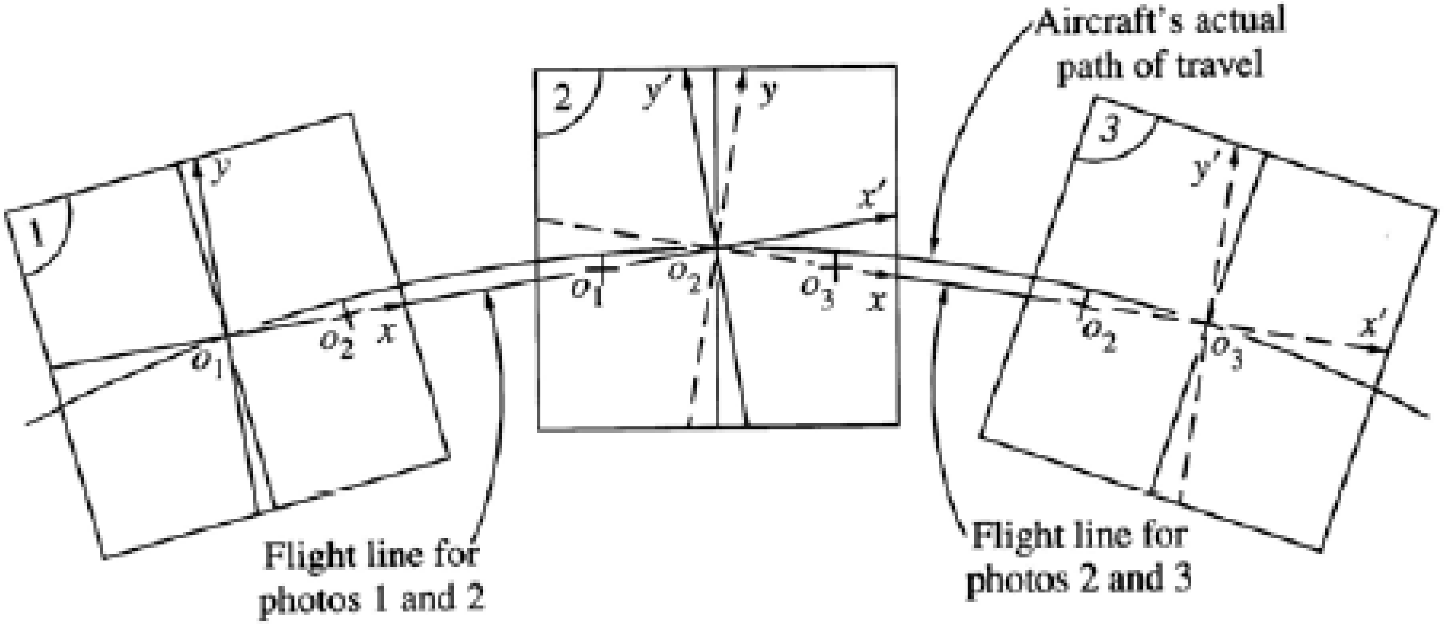
$L_1$
$L_2$
o
b
a
b'
a'
o'
A
B



L1, L2
pb
pa
b'
o, o'
a'
b
a
x'b
x'a
xb
xa

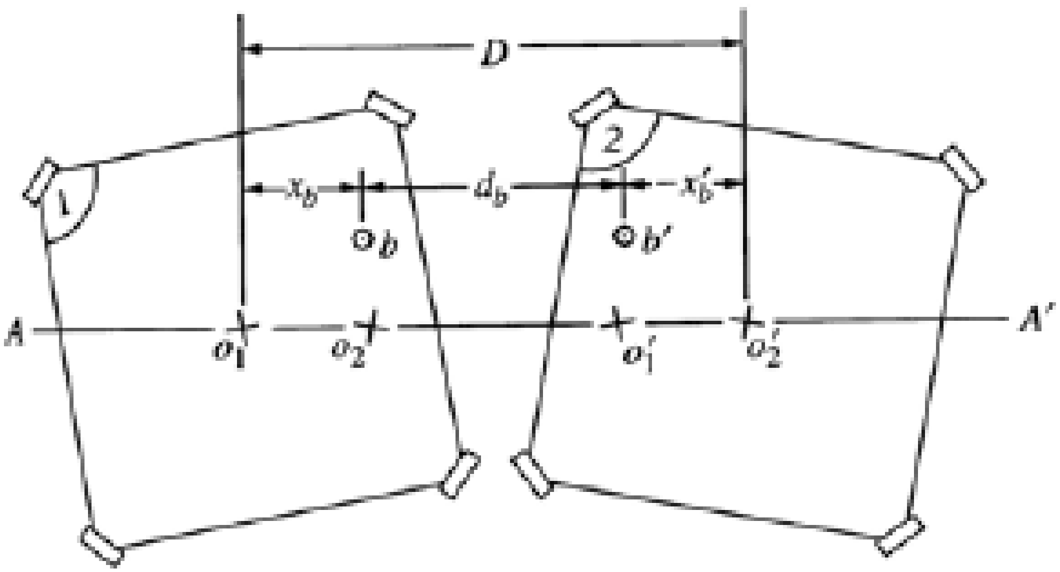
# STEREOSCOPIC PARALLAX

- Parallax measured in flight-line axis system



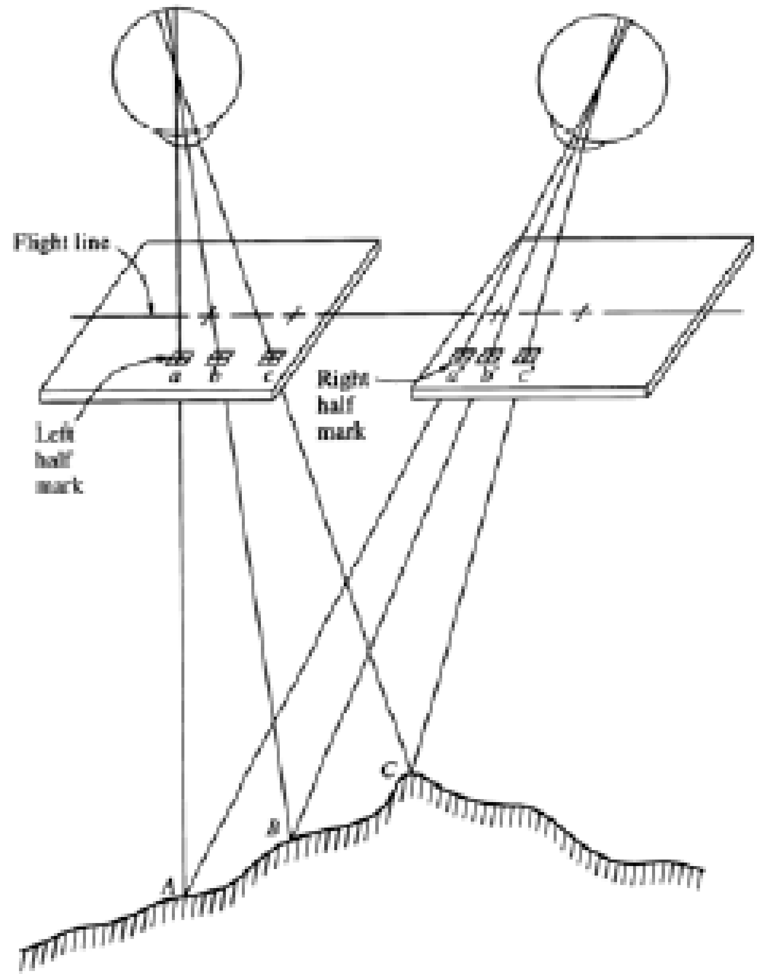
# MONOSCOPIC PARALLAX MEASUREMENT

- Mark conjugate principal points
- Align flight line axis
- Parallax: $p_b = D - d_b$



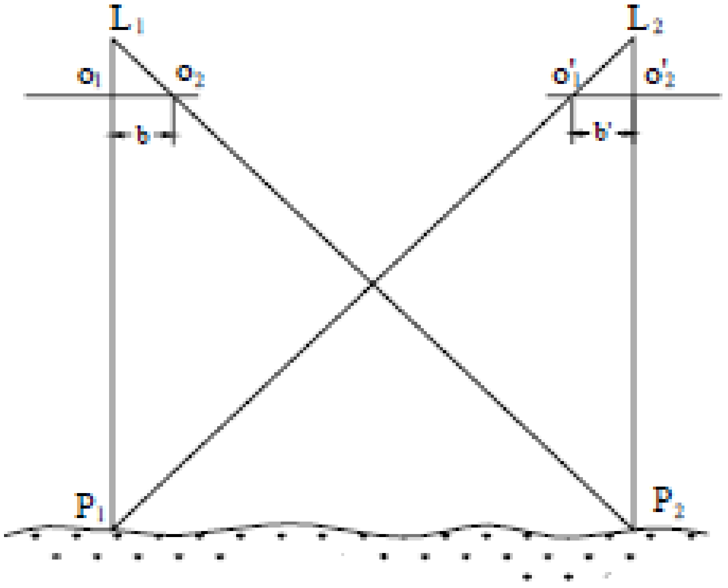
# PRINCIPLE OF FLOATING MARK

- When viewing in stereo, 2 small identical marks etched on clear glass
  - Called half marks
- Half marks shifted until they fuse into single mark
- If marks moved closer together, they appear to rise
- If moved apart, marks appear to fall
- Spacing of half marks, hence their parallax, varied so floating marks appears to rest exactly on terrain

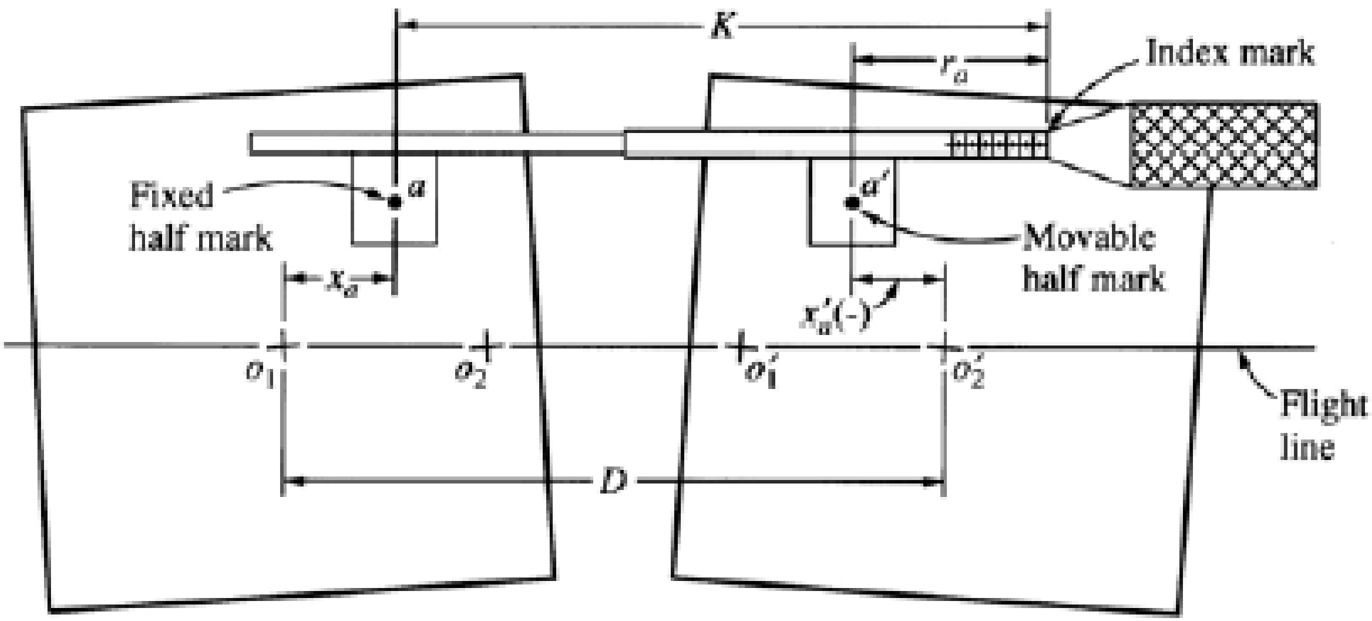


# PARALLAX OF PRINCIPAL POINT

- Parallax of left ground principal point is photo base b' measured on right photo, and the parallax of right ground principal point is photo base b measured on left photo
- For moderate relief, $b \approx b'$
- Photo base is average of two values



# STEREOSCOPIC PARALLAX MEASUREMENT



# STEREOSCOPIC PARALLAX MEASUREMENT

- Parallax bar measurement

$$p_a = x_a - x'_a = D - (K - r_a) = (D - K) + r_a$$

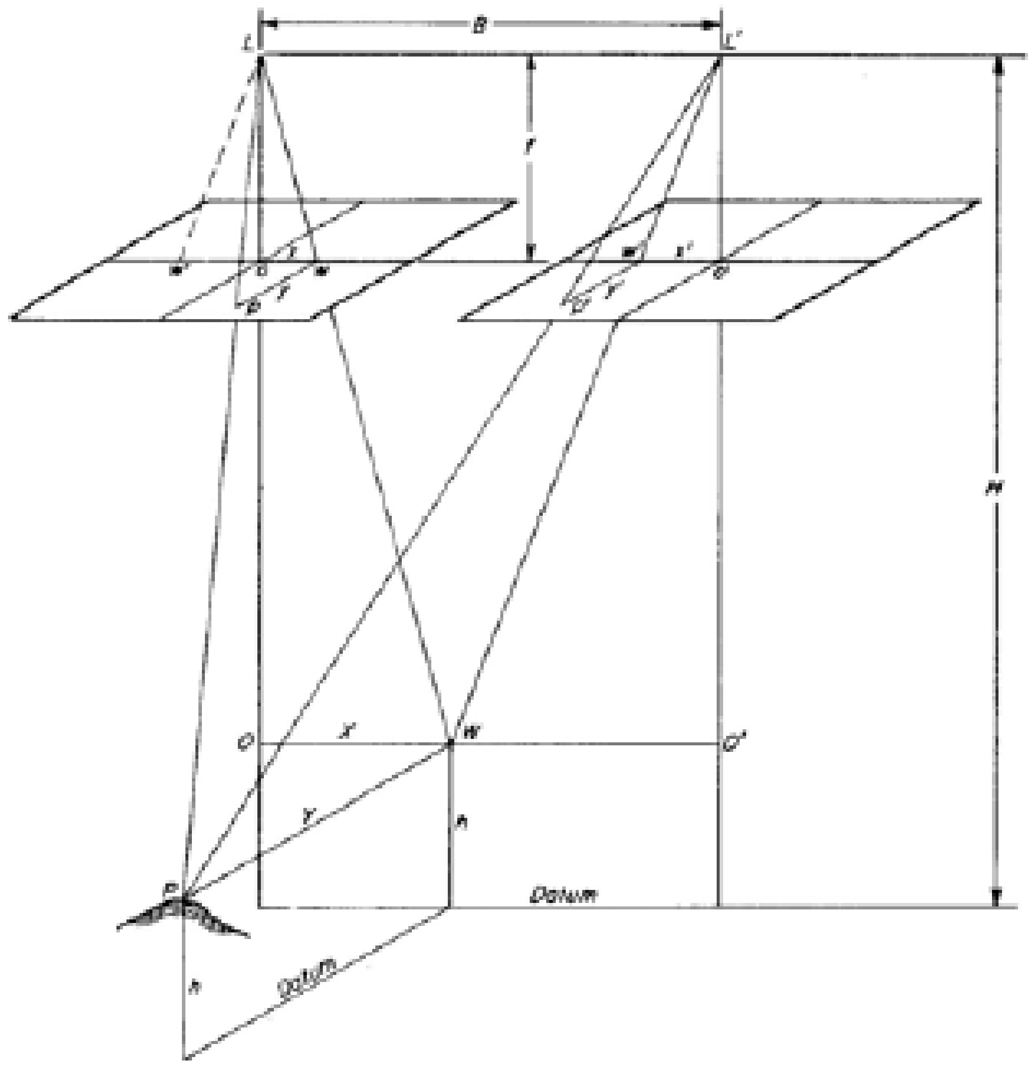
- Substituting parallax bar constant C

$$p_a = C + r_a$$

- To compute C, measure parallax monoscipically and take micrometer reading

$$C = p - r$$

# DEVELOPMENT OF PARALLAX EQUATIONS



# DEVELOPMENT OF PARALLAX EQUATIONS

- Triangles LOW and Low, write scale:

$$\frac{Lo}{LO} = \frac{Lw}{LW} = \frac{ow}{OW} \quad \Rightarrow \quad \frac{f}{H-h} = \frac{x}{X}$$

- From triangles Lwp and LWP, write scale:

$$\frac{Lw}{LW} = \frac{wp}{WP} = \frac{f}{H-h} = \frac{y}{Y}$$

# DEVELOPMENT OF PARALLAX EQUATIONS

- Using triangles L'O'W, L'o'w', L'w'p', L'WP

$$\frac{L'o'}{L'O'} = \frac{L'w'}{L'W} = \frac{w'p'}{WP} = \frac{y'}{Y} \quad \Rightarrow \quad \frac{y'}{Y} = \frac{f}{H-h}$$

- From last two relationships:

$$\frac{y}{Y} = \frac{f}{H-h} \qquad\qquad \frac{y'}{Y} = \frac{f}{H-h}$$

- Yielding:

$$y = y'$$

# DEVELOPMENT OF PARALLAX EQUATIONS

- In triangles LWL' & Lww'
  - LL' is parallel to ww'
  - LW is parallel to Lw
  - L'W is parallel to Lw'
  - The two triangles are similar triangles
  - Corresponding altitudes are (H – h) and f

# DEVELOPMENT OF PARALLAX EQUATIONS

- From similar triangles

$$\frac{f}{H-h} = \frac{ww'}{B}$$

- Since

$$ww' = x - x' = p$$

- then

$$\frac{f}{H-h} = \frac{p}{B}$$

# DEVELOPMENT OF PARALLAX EQUATIONS

- The parallax equations are:

$$H - h = \frac{B}{p} f \quad \Rightarrow \quad h = H - \frac{Bf}{p}$$

$$X = \frac{B}{p} x$$

$$Y = \frac{B}{p} y$$

# DEVELOPMENT OF PARALLAX EQUATIONS

- The parallax equations are:

$$H - h = \frac{B}{p} f \quad \Rightarrow \quad h = H - \frac{Bf}{p}$$

$$X = \frac{B}{p} x$$

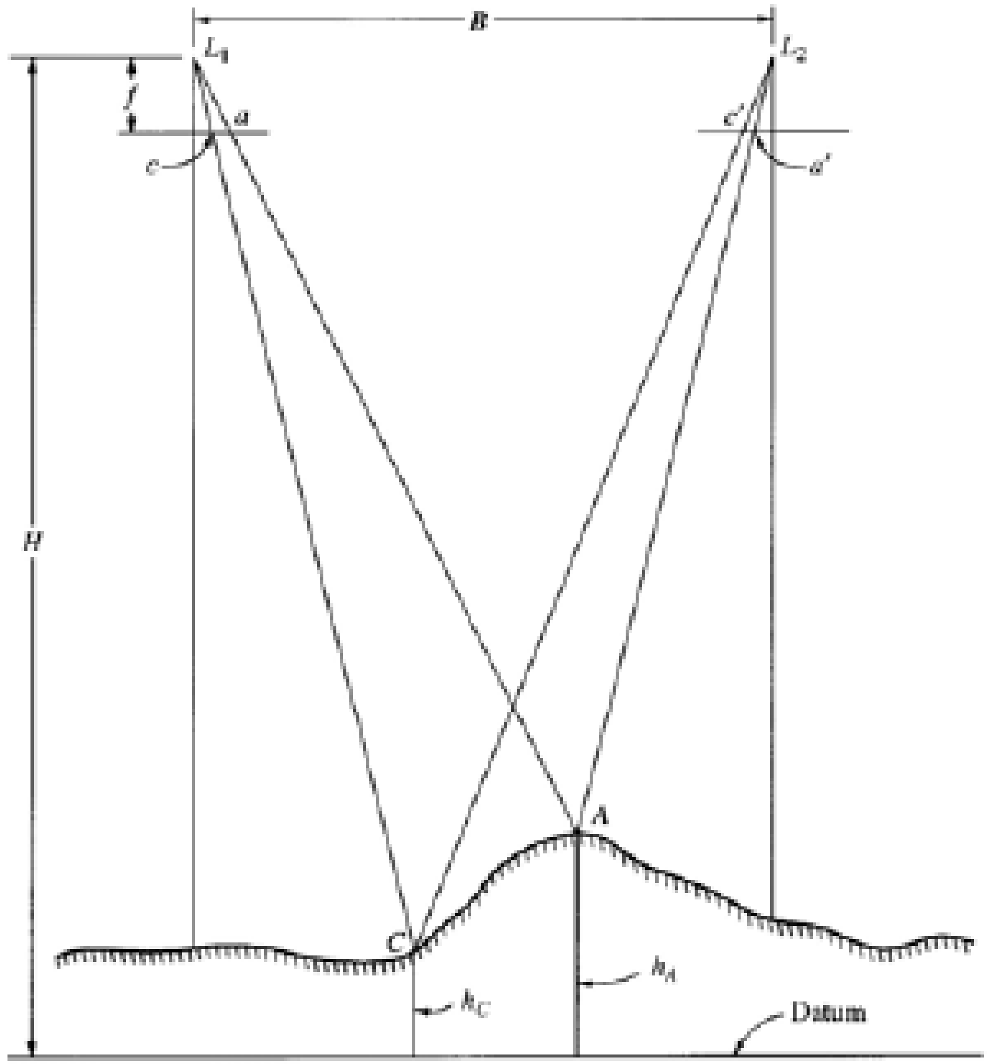
$$Y = \frac{B}{p} y$$

# PARALLAX EQUATIONS

- Valid for
  - Truly vertical photographs only
  - Photos taken from same flying height
  - Coordinates (x, y, x', y') related to flight line axis system

- Ground coordinates not related to true ground coordinates but to the coordinate system of the stereopair

# ELEVATION BY PARALLAX DIFFERENCES



# ELEVATION BY PARALLAX DIFFERENCES

- Recall parallax formula:

$$h = H - \frac{Bf}{p}$$

- Rearrange for points a and c

$$p_c = \frac{fB}{H - h_C} \qquad p_a = \frac{fB}{H - h_A}$$

# ELEVATION BY PARALLAX DIFFERENCES

Parallax difference:

$$\Delta p = p_a - p_c$$

$$= \frac{fB}{H - h_A} - \frac{fB}{H - h_C}$$

$$= \frac{fB\left(H - h_C\right) - fB\left(H - h_A\right)}{\left(H - h_A\right)\left(H - h_C\right)}$$

$$= \frac{fB\left(h_A - h_C\right)}{\left(H - h_A\right)\left(H - h_C\right)}$$

# ELEVATION BY PARALLAX DIFFERENCES

- Substituting parallax formula for flying height above the terrain

$$\Delta p = \frac{fB\left(h_A - h_C\right)}{\left(\frac{fB}{p_a}\right)\left(H - h_C\right)}$$

$$= \frac{p_a\left(h_A - h_C\right)}{H - h_C}$$

- From which $h_A = h_C + \dfrac{\Delta p\left(H - h_C\right)}{p_a}$

# ELEVATION BY PARALLAX DIFFERENCES

- Alternative development

$$\Delta h = h_A - h_C$$

$$= \left( H - \frac{Bf}{p_a} \right) - \left( H - \frac{Bf}{p_c} \right)$$

$$= \frac{Bf\Delta p}{p_c(p_c + \Delta p)}$$

# ELEVATION BY PARALLAX DIFFERENCES

- Since ground principal points lie on same datum

$$o_1 o_2 = o'_1 \, o'_2 = b$$

- Since $O_1$, $O_2$ and C lie at same elevation, their parallaxes are the same

$$p_c = b$$

# ELEVATION BY PARALLAX DIFFERENCES

- From figure

$$\frac{b}{B} = \frac{f}{H - h_c}$$

- From which

$$B = \frac{(H - h_c)\,b}{f}$$

# ELEVATION BY PARALLAX DIFFERENCES

- Substitute in elevation difference formula and recognizing that $p_c$ = b

$$\Delta h = \frac{(H - h_c)\Delta p}{b + \Delta p}$$

# ERROR EVALUATION

- Some sources of errors
  - Locating and marking flight lines
  - Orienting stereopairs for parallax measurements
  - Parallax and photo coordinate measurements
  - Shrinkage or expansion of photos
  - Unequal flying heights
  - Tilted photographs
  - Errors in ground control
  - Other errors: camera lens distortion, atmospheric refraction distortion

# ERROR EVALUATION

- General approach – differentiate equation
- Example for basic parallax equations

$$h = H - \frac{Bf}{p}$$

$$\frac{\partial h}{\partial H} = 1 \quad ; \quad \frac{\partial h}{\partial B} = -\frac{f}{p} \quad ; \quad \frac{\partial h}{\partial p} = \frac{Bf}{p^2}$$