Analytical Photogrammetry

Exterior Orientation

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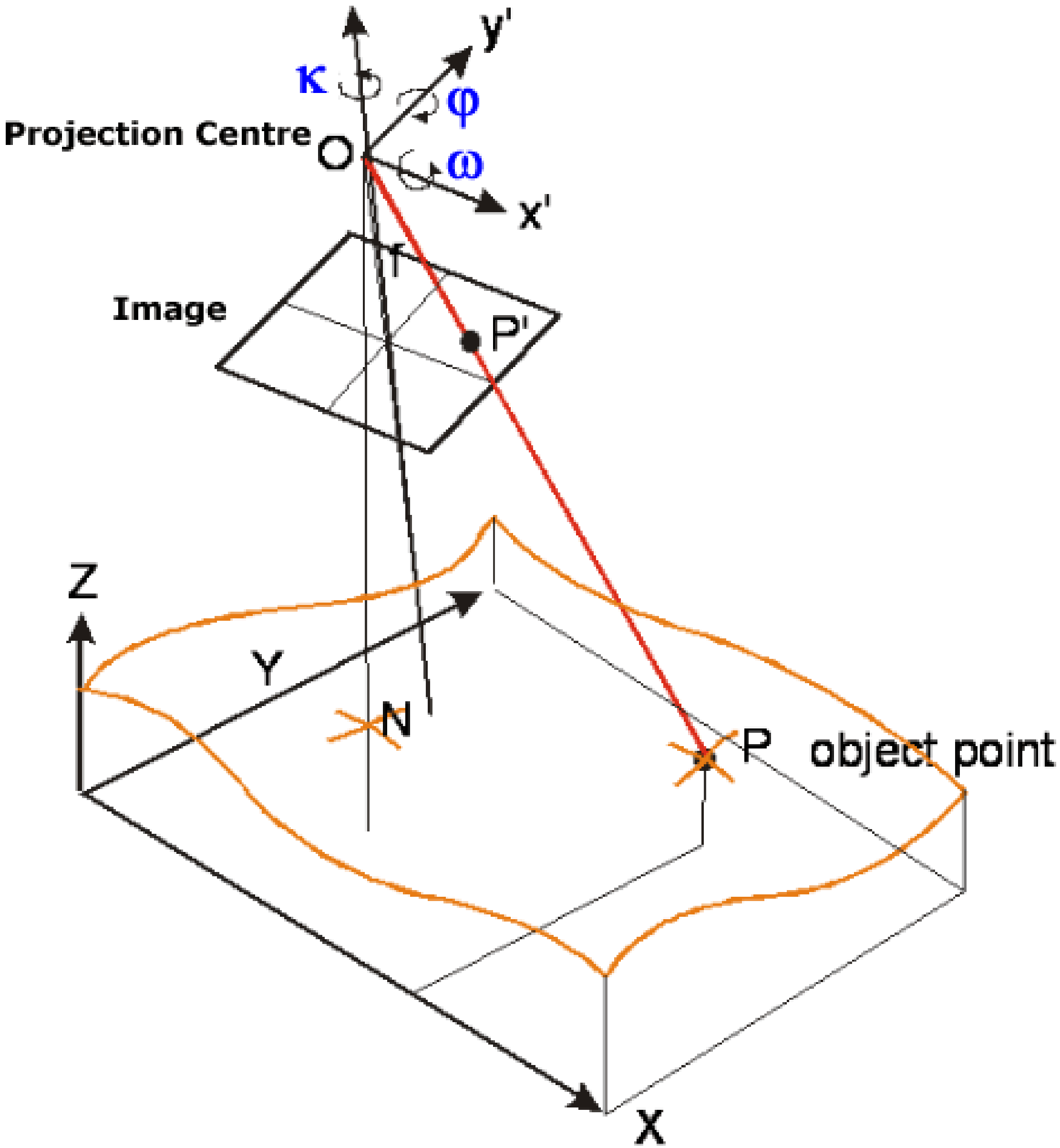


Exterior Orientation

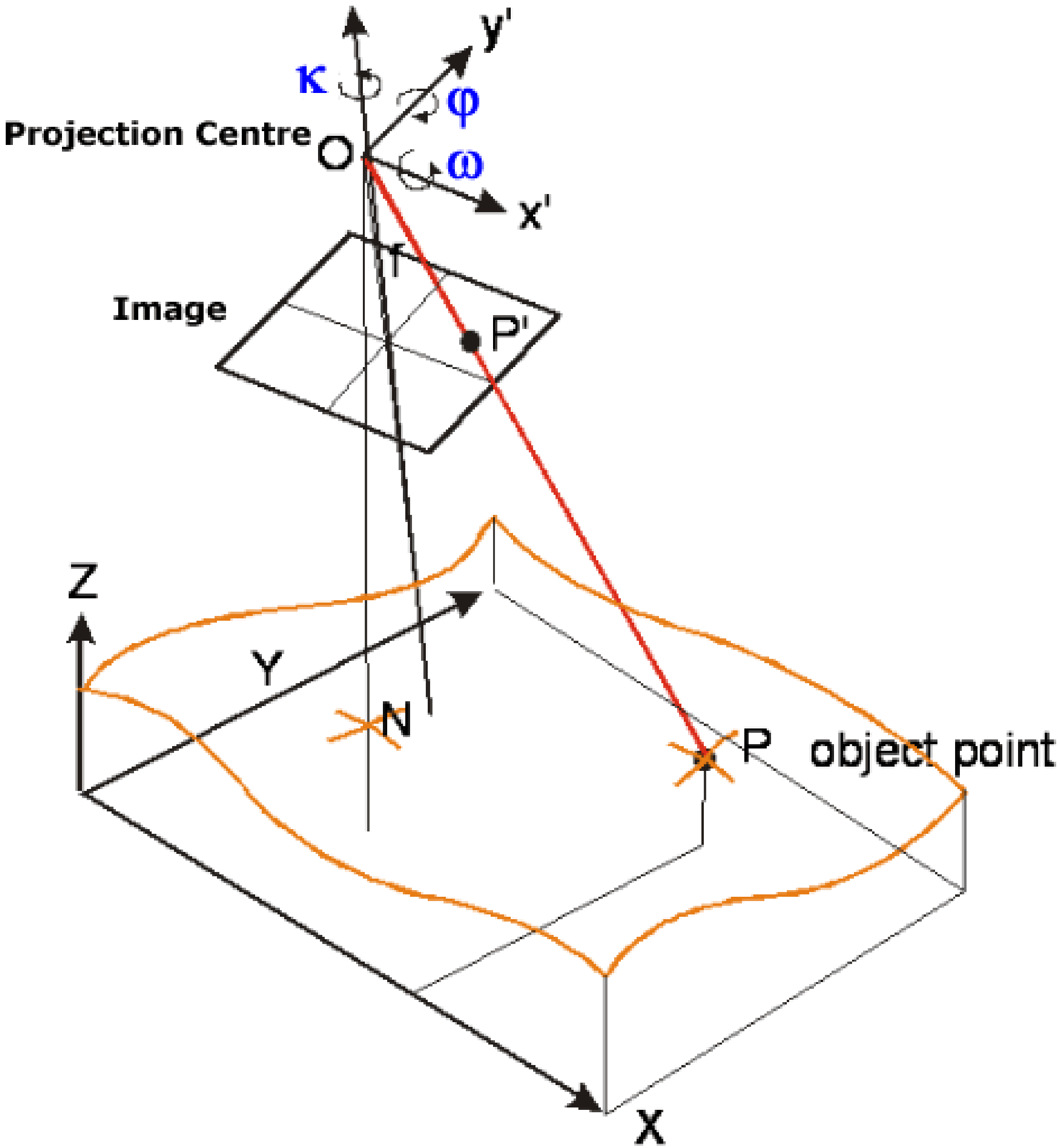
- Exterior orientation is the relationship between image and object space.
- The camera position in the object coordinate system must be determined by the *location* of its perspective center and by its *attitude*, expressed by three independent angles.

Exterior Orientation

- *Determine* position and orientation of camera in absolute coordinate system from projections of calibration points in scene
- The exterior orientation of the camera is specified by all parameters of camera pose, such as perspectivity center position, optical axis direction.
- Exterior orientation specification: requires 3 rotation angles, 3 translations

- The problem of establishing the six orientation parameters of the camera can conveniently be solved by the collinearity model.
- This model expresses the condition that the perspective center *C, the image point Pi, and the object point Po, must lie on a straight* line.

Exterior Orientation



- If the exterior orientation is known, then the image vector **p*i*** and the

vector **q** in object space are collinear:

$$p_i = \frac{1}{\lambda} q \tag{5.19}$$

- As depicted in Fig., vector **q** is the difference between the two point vectors **c**

and **p.** For satisfying the collinearity condition, we rotate and scale **q** from object to

image space. We have

$$p_i = \frac{1}{\lambda} R\, q = \frac{1}{\lambda} R \left(p - c \right) \tag{5.20}$$

- with **R** an orthogonal rotation matrix with the three angles *ω, φ and κ:*

$$\mathrm{R} = \begin{vmatrix} \cos\phi\cos\kappa & -\cos\phi\sin\kappa & \sin\phi \\ \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & -\sin\omega\cos\phi \\ \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa & \cos\omega\cos\phi \end{vmatrix} \tag{5.21}$$

Eq.5.20 renders the following coordinate equations

$$\begin{aligned} x &= \frac{1}{\lambda}(X_P - X_C)r_{11} + (Y_P - Y_C)r_{12} + (Z_P - Z_C)r_{13} &\quad (5.22)\\ y &= \frac{1}{\lambda}(X_P - X_C)r_{21} + (Y_P - Y_C)r_{22} + (Z_P - Z_C)r_{23} &\quad (5.23)\\ -c &= \frac{1}{\lambda}(X_P - X_C)r_{31} + (Y_P - Y_C)r_{32} + (Z_P - Z_C)r_{33} &\quad (5.24) \end{aligned}$$

By dividing the first by the third and the second by the third equation, the scale factor $1/\lambda$ *is eliminated leading to the following two collinearity equations:*

$$x = -c\frac{(X_P - X_C)r_{11} + (Y_P - Y_C)r_{12} + (Z_P - Z_C)r_{13}}{(X_P - X_C)r_{31} + (Y_P - Y_C)r_{32} + (Z_P - Z_C)r_{33}} \qquad (5.25)$$

$$y = -c\frac{(X_P - X_C)r_{21} + (Y_P - Y_C)r_{22} + (Z_P - Z_C)r_{23}}{(X_P - X_C)r_{31} + (Y_P - Y_C)r_{32} + (Z_P - Z_C)r_{33}} \qquad (5.26)$$

with:

$$\mathrm{p}_i = \begin{bmatrix} x \\ y \\ -f \end{bmatrix} \qquad \mathrm{p} = \begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} \qquad \mathrm{c} = \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}$$

The six parameters: *XC, YC, ZC, ω, φ, κ are the unknown elements of exterior orientation.*
The image coordinates *x, y are normally known (measured) and the calibrated* focal length *c is a constant.*

- Every measured point leads to two equations, but also adds
- three other unknowns, namely the coordinates of the object point (*XP , YP, ZP). Unless*
- the object points are known (control points), the problem cannot be solved with only
- one photograph.

- The collinearity model as presented here can be expanded to include parameters of the interior orientation.
- The number of unknowns will be increased by three (*Parameters of interior orientation*: position of principal point and calibrated focal length).
- This combined approach lets us determine simultaneously the parameters of interior and exterior orientation of the cameras.

Single Photo Resection

- The position and attitude of the camera with respect to the object coordinate system (exterior orientation of camera) can be determined with help of the collinearity equations.
- Eqs. 5.27 express measured quantities as a function of the exterior orientation parameters.

Thus, the collinearity equations can be directly used as observation equations, as the following functional representation illustrates.

$$x, y = f(\underbrace{X_C, Y_C, Z_C, \omega, \phi, \kappa}_{\text{exterior orientation}}, \underbrace{X_P, Y_P, Z_P}_{\text{object point}}) \qquad (5.27)$$

- For every measured point two equations are obtained. If three control points are measured, a total of 6 equations is formed to solve for the 6 parameters of exterior orientation.

- The collinearity equations are not linear in the parameters.
- Therefore, Eqs. 5.25 and 5.26 must be linearized with respect to the parameters.
- This also requires approximate

  values with which the iterative process will start.

The image-to-ground coordinate relationship is established through the collinearity model and is represented by the collinearity equations:

$$x = x_p - c\frac{r_{11}\cdot(X - X_o) + r_{21}\cdot(Y - Y_o) + r_{31}\cdot(Z - Z_o)}{r_{13}\cdot(X - X_o) + r_{23}\cdot(Y - Y_o) + r_{33}\cdot(Z - Z_o)}$$

$$y = y_p - c\frac{r_{12}\cdot(X - X_o) + r_{22}\cdot(Y - Y_o) + r_{32}\cdot(Z - Z_o)}{r_{13}\cdot(X - X_o) + r_{23}\cdot(Y - Y_o) + r_{33}\cdot(Z - Z_o)}$$

The above equations involve the following quantities:

- The measured image point coordinates: x, y
- Interior orientation parameters of the camera: x_p, y_p, c
- Exterior orientation parameters of the image under consideration: X_o, Y_o, Z_o, ω, ϕ, κ

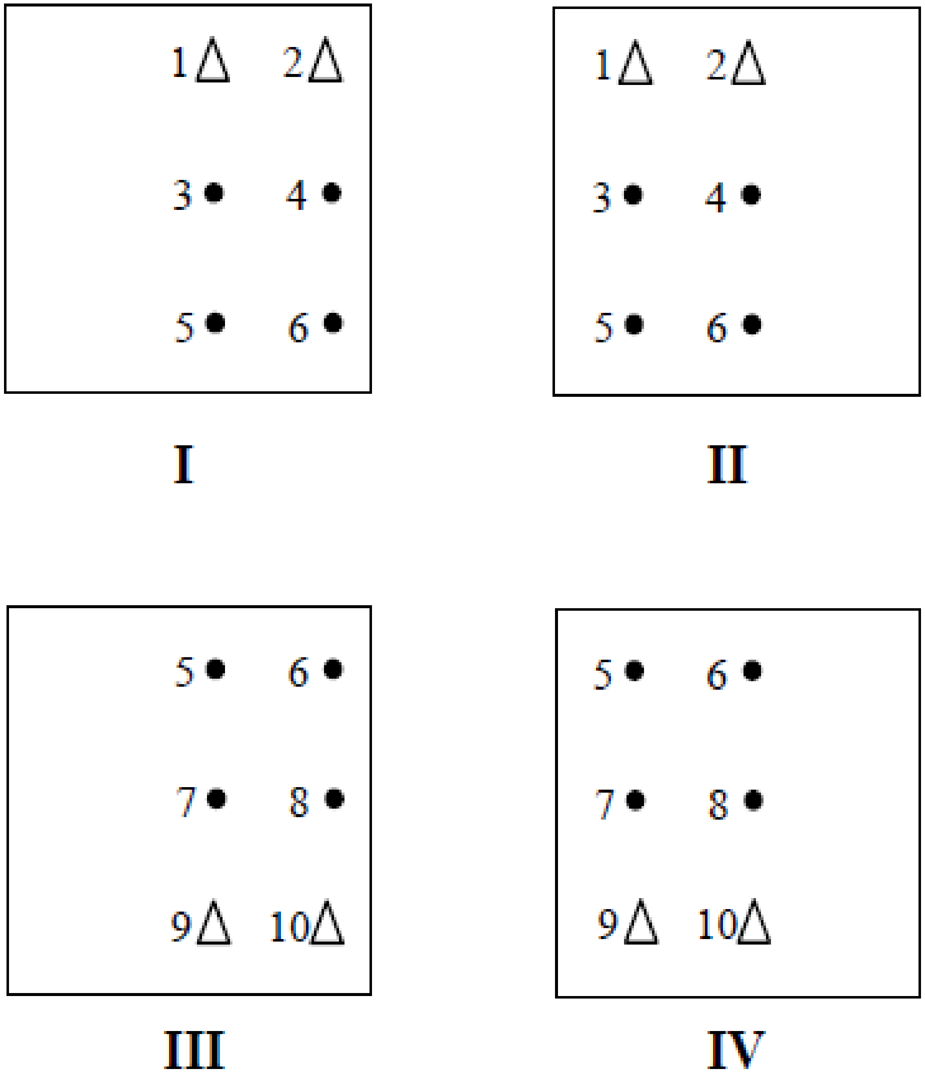
  where ω, ϕ, κ are embedded in the rotation matrix components:

$r_{11} = \cos\phi \cos\kappa$	$r_{12} = -\cos\phi \sin\kappa$	$r_{13} = \sin\phi$
$r_{21} = \cos\omega \sin\kappa + \sin\omega \sin\phi \cos\kappa$	$r_{22} = \cos\omega \cos\kappa - \sin\omega \sin\phi \sin\kappa$	$r_{23} = -\sin\omega \cos\phi$
$r_{31} = \sin\omega \sin\kappa - \cos\omega \sin\phi \cos\kappa$	$r_{32} = \sin\omega \cos\kappa + \cos\omega \sin\phi \sin\kappa$	$r_{33} = \cos\omega \cos\phi$

- The ground coordinates of point: X, Y, Z.

Given:

Four images in two strips



Points 1, 2, 9, 10 are Full Control Points

Points 3, 4, 5, 6, 7, 8 are tie points (i.e., they appear in more than one image and have unknown ground coordinates)

- **Required:**
- we need to solve for the EOP's of the four images and the ground coordinates of tie points assuming
- known/errorless IOP's and ground coordinates of control points, analyze the observations-parameters and show the structure of each component of the *Observation Equation Model* (*Gauss-Markov model*) required to perform a least squares adjustment procedure;
- $y = Ax + e$.

- **Observation Equation Model:**

$$y = A\,x + e$$

where,

y: vector of observations,

A: the design matrix,

x : vector of parameters,

e: vector of errors.

SOLUTION

- **Observation-parameters analysis:**
- a) The observations
- The measured image coordinates of points 1,2,3,4,5,6 on image I   6 x 2 =12
- The measured image coordinates of points 1,2,3,4,5,6 on image II   6 x 2 =12
- The measured image coordinates of points 5,6,7,8,9,10 on image III   6 x 2 =12
- The measured image coordinates of points 5,6,7,8,9,10 on image IV   6 x 2 =12
- **Total n = 48**

- b) The parameters/unknowns
- Known interior orientation
  parameters for the camera: xp, yp, c — 0 (erroless)
- Exterior orientation parameters:
  Xo, Yo, Zo, ω, ϕ, κ for images I,II,III,IV — 4 x 6 =24
- The ground coordinates of the control
  points 1,2,9,10 — 0 (erroless)
- The ground coordinates of the
  tie points 3,4,5,6,7,8 — 6 x 3 =18
- **Total m = 42**

- c) Redundancy
- $n - m = 48 - 42 = 6$.
- Due to this redundancy, the unknown parameters are determined through a least squares adjustment procedure.