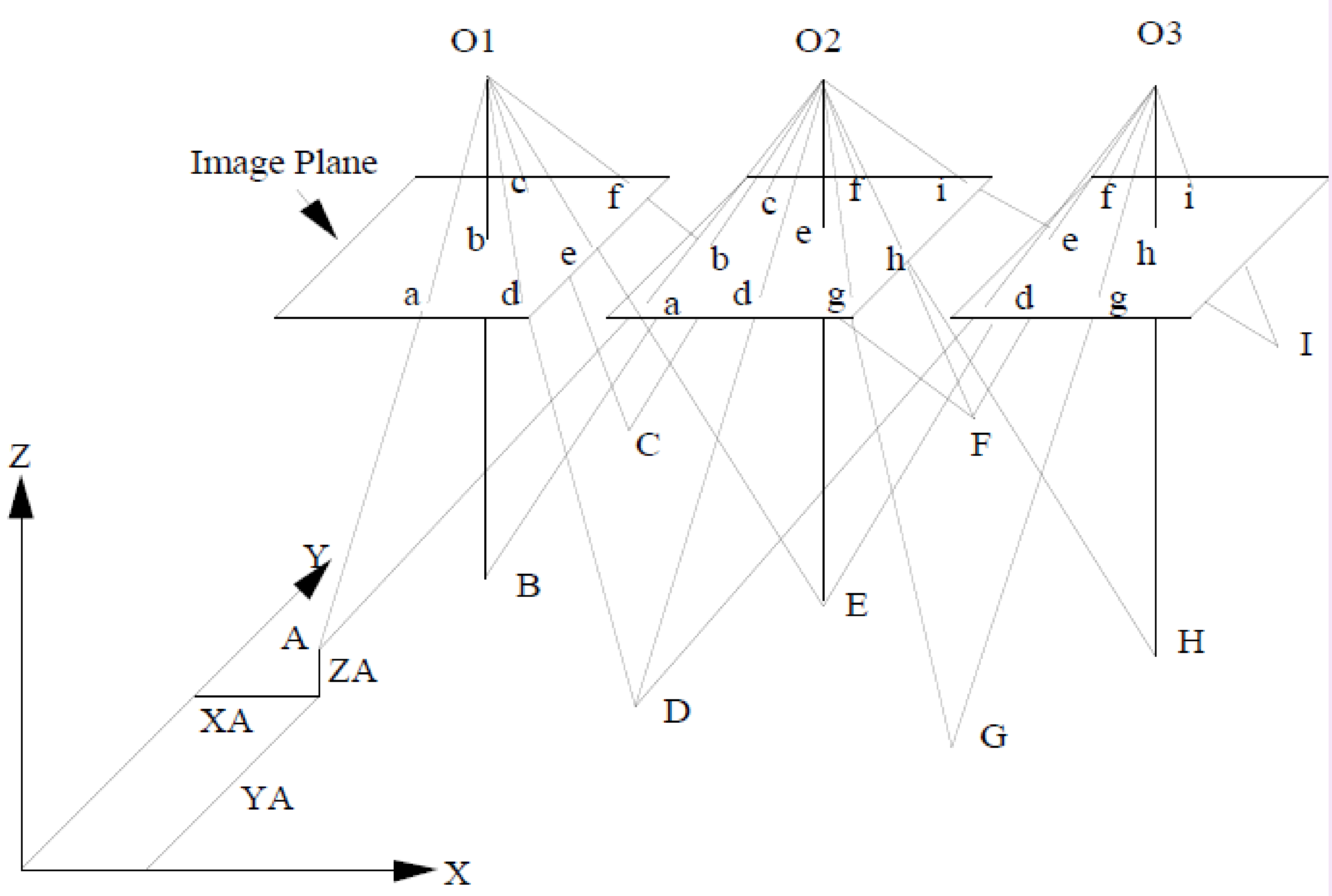
Aerial Triangulation

PURPOSE

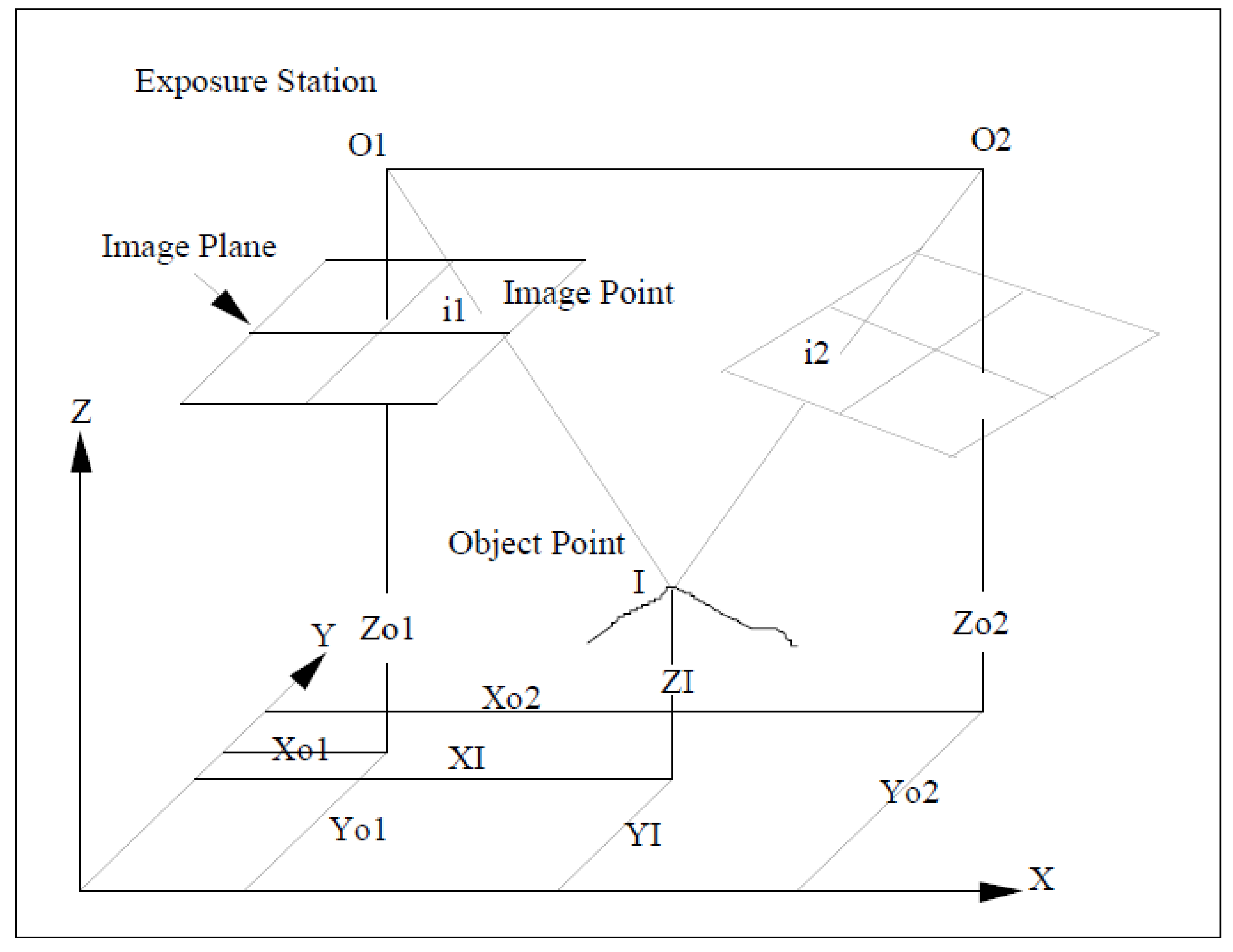
- Main purpose of aerial triangulation (AT) is the determination of ground coordinates for a large number of terrain points and
- the exterior orientation parameters of aerial photographs using as few control points as possible.

- The integration of GPS measurements into photogrammetric blocks allows the
- accurate determination of coordinates of the exposure stations resulting in a reduction of
- the number of ground control points to a minimum.

BUNDLE BLOCKS



Equation of aerial triangulation: The Collinearity Equation



Colllinearity equations

$$F_x = x_i - x_o + c\frac{m_{11}(X_i - X_O) + m_{12}(Y_i - Y_O) + m_{13}(Z_i - Z_O)}{m_{31}(X_i - X_O) + m_{32}(Y_i - Y_O) + m_{33}(Z_i - Z_O)} = 0$$

$$F_y = y_i - y_o + c\,k_y\frac{m_{21}(X_i - X_O) + m_{22}(Y_i - Y_O) + m_{23}(Z_i - Z_O)}{m_{31}(X_i - X_O) + m_{32}(Y_i - Y_O) + m_{33}(Z_i - Z_O)} = 0$$

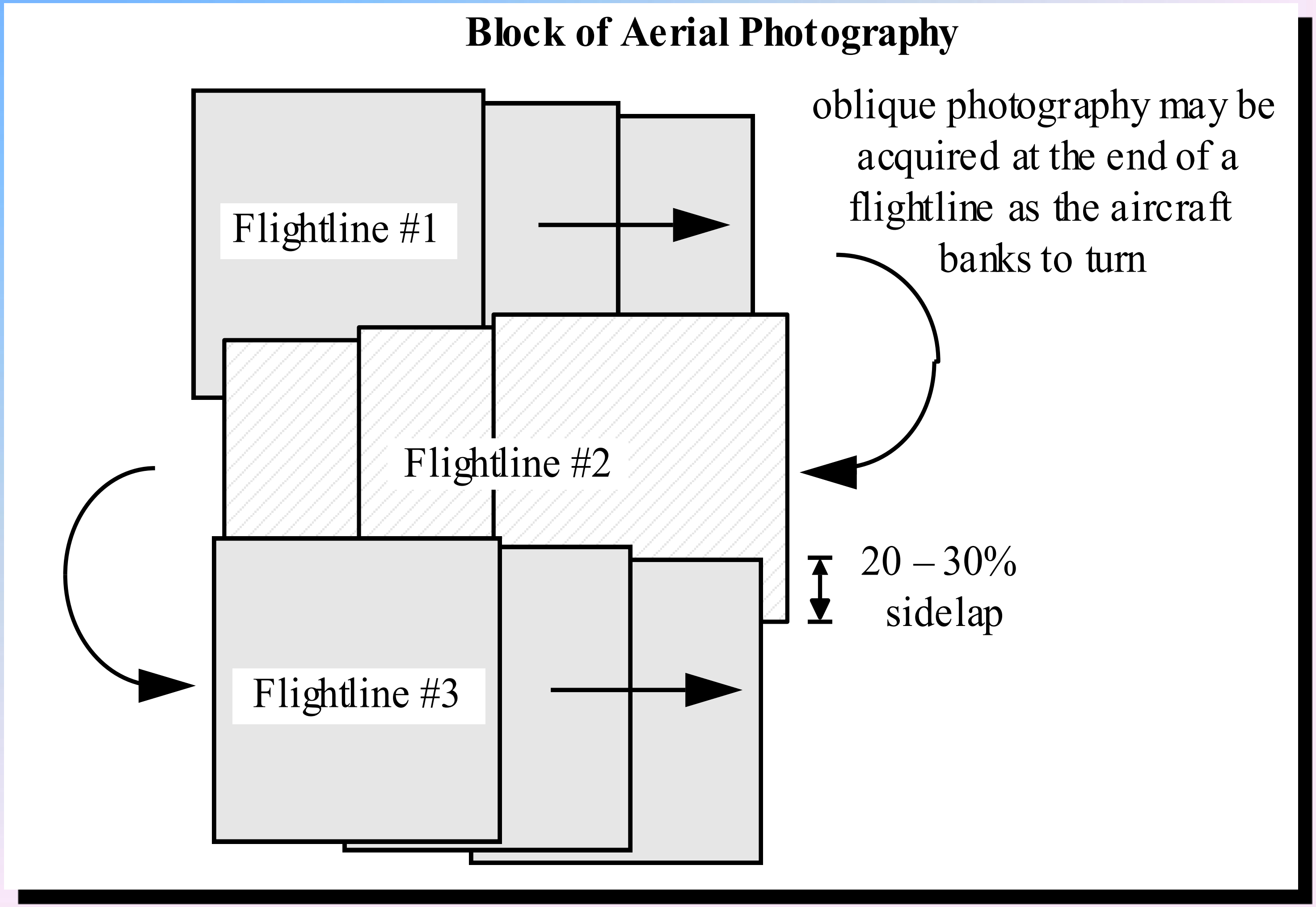
$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\Phi)\cos(\kappa) & \cos(\omega)\cos(\kappa)+\sin(\omega)\sin(\Phi)\cos(\kappa) & \sin(\omega)\sin(\kappa)-\cos(\omega)\sin(\Phi)\cos(\kappa) \\ -\cos(\Phi)\sin(\kappa) & \cos(\omega)\cos(\kappa)-\sin(\omega)\sin(\Phi)\sin(\kappa) & \sin(\omega)\cos(\kappa)+\cos(\omega)\sin(\Phi)\sin(\kappa) \\ \sin(\Phi) & -\sin(\omega)\cos(\Phi) & \cos(\omega)\cos(\Phi) \end{bmatrix}$$

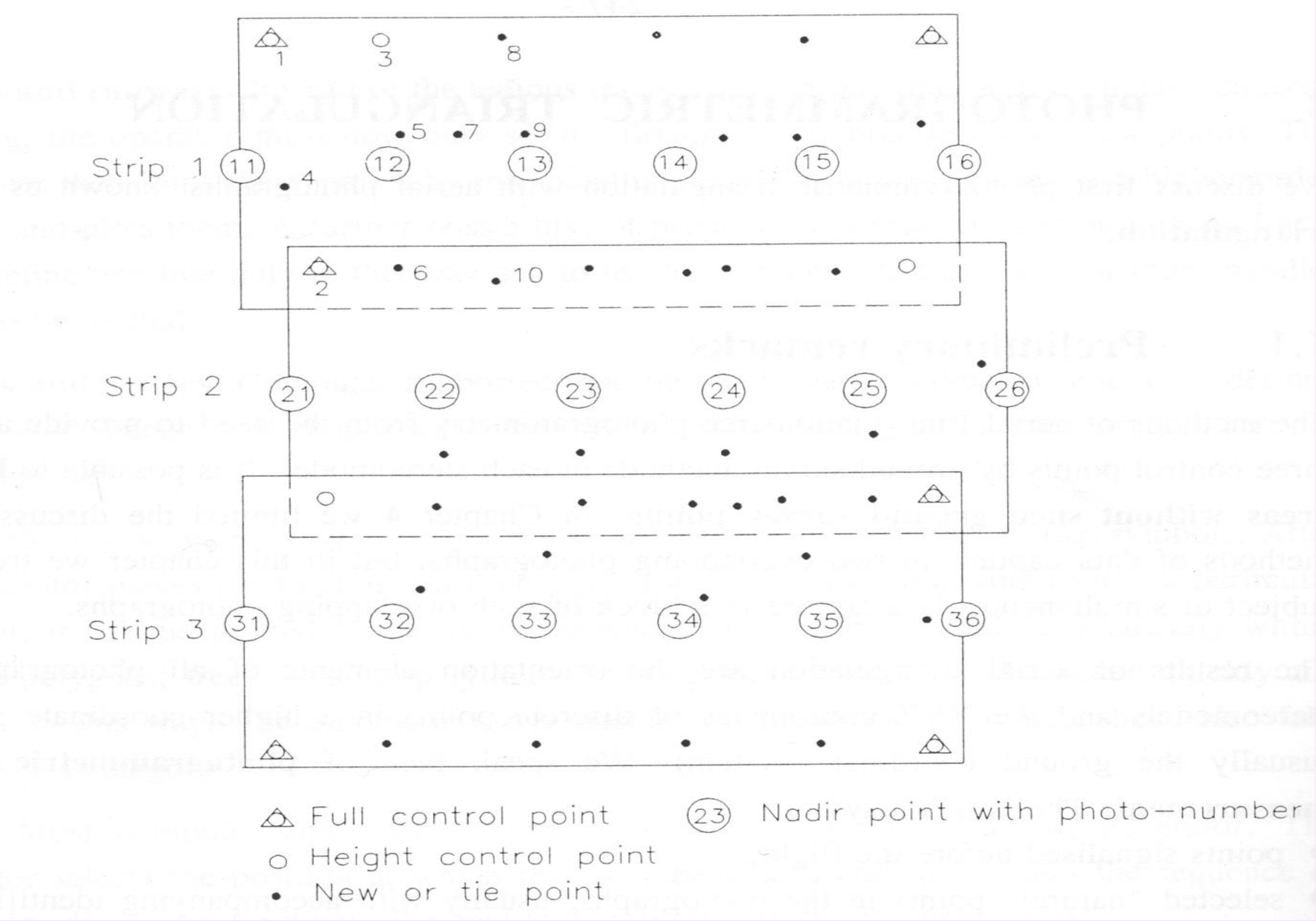
- (x_i, y_i) are the image coordinates,
- (x_o, y_o) are the principal point coordinates,
- c is the camera constant,
- m_{ij} is an element of the rotation matrix,
- X_i, Y_i, Z_i are the object point coordinates,
- X_O, Y_O, Z_O are the exposure station coordinates,
- M is the rotation matrix,
- k_y is the scale factor for y axis in digital camera (this factor is 1 for film based camera).

- There are 6 unknowns in the collinearity equations, namely, the exterior orientation parameters XO,YO,ZO,ω ,θ ,κ . The three rotation angles ω ,θ ,κ are implicit in the rotation matrix M .
- The principal point coordinates (xo , yo) and camera constant (c) are considered to be known for the basic bundle approach.

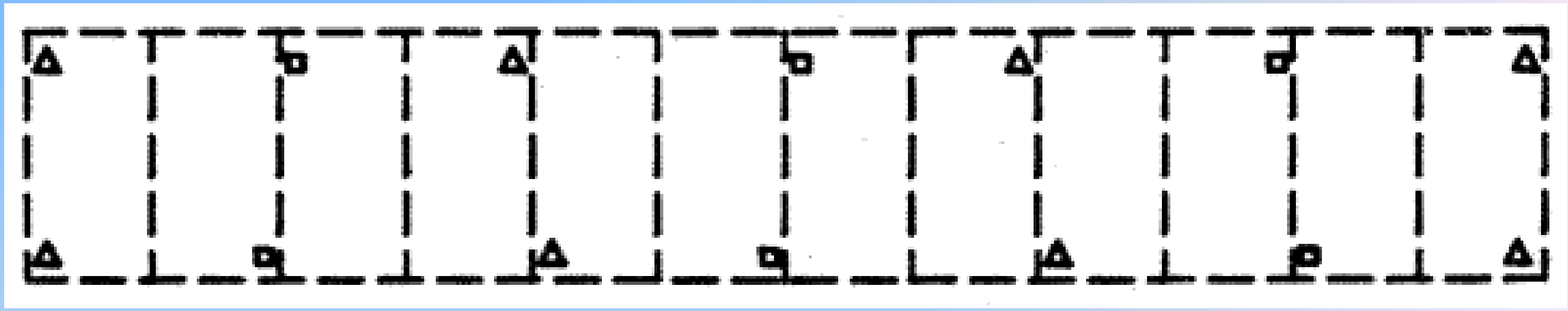
Block of Vertical Aerial Photography



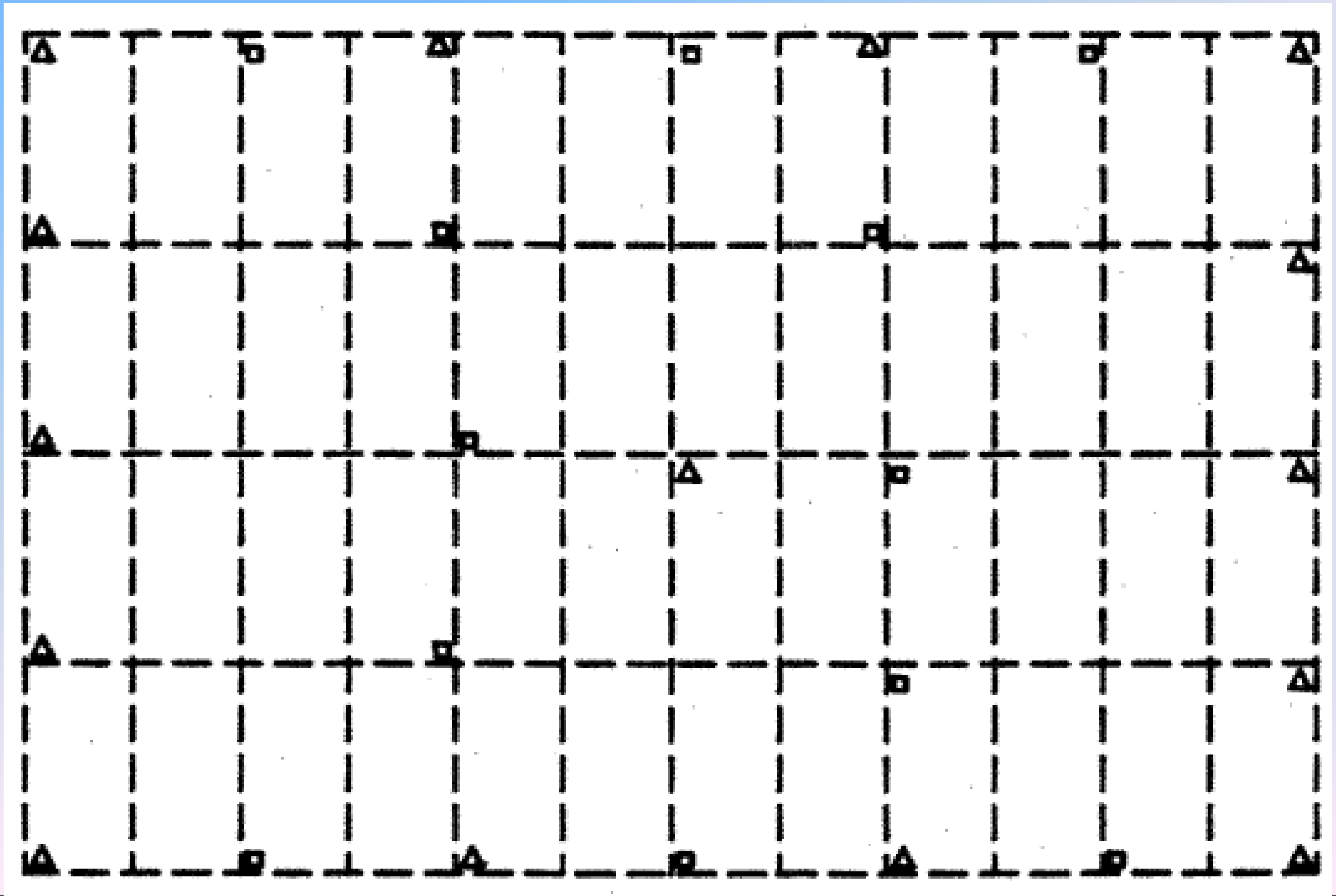
Discret Points



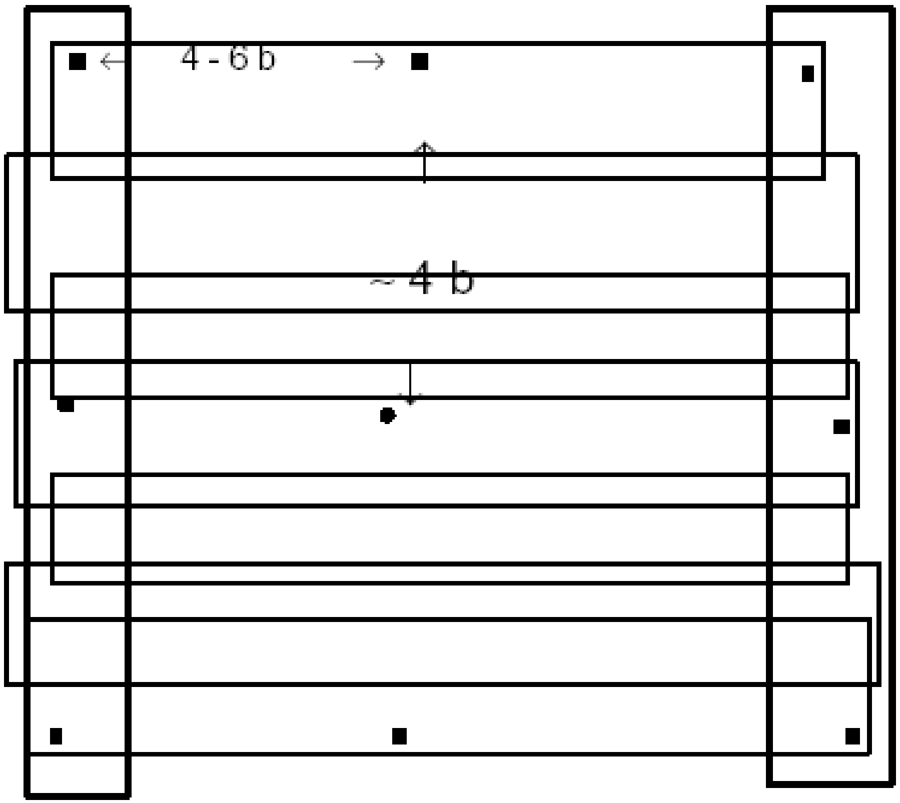
Control Points



Control Points for blocks



Control Points



- vertical control point
- X,Y,Z-control point

control point distribution for blocks with crossing strips

Block adjustment by independent models

Block adjustment by independent models

- The adjustment of the block by independent models begins with the model co-ordinates derived from the numerical relative orientations and formation of the streomodels.
- These streomodels are brought together into a single block and simultaneouosly transformed into the ground coordinate system.
- The individual steromodels are the units of aerial triangulation by independent models.

Block adjustment by independent models

- It involves manual relative orientation in a stereoplotter of each stereo model of a strip or block of photos. After the models have been measured, they are numerically adjusted to the ground system by either a sequential or a simultaneous method.

Block adjustment by independent models

- Planimetric Adjustment of a Block
- Spatial Block Adjustment

Planimetric Adjustment of a Block

- The adjustment of planimetry concerns only with the XY co-ordinates.
- As result we want to obtain the XY co-ordinates of new points in the ground co-ordinate system.

- The model co-ordinates x,y of the relatively oriented and leveled individual models are given values.
- The corresponding image co-ordinates of these points are measured and used to compute the model co-ordinates after the relative orientation.
- With the help of these points the numerical absolute orientation can be performed to level the model.

Planimetric Adjustment of a Block

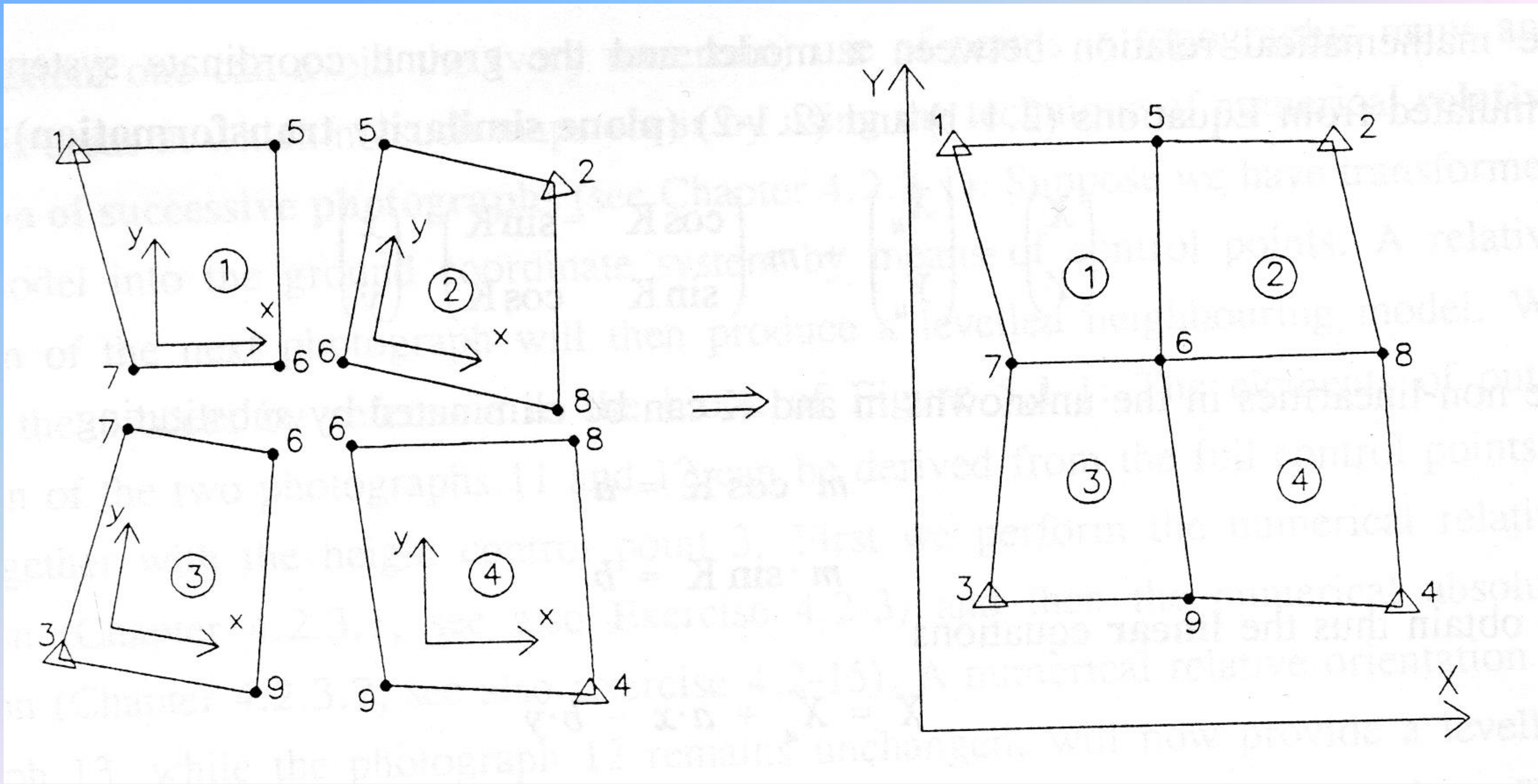
- <u>Unknowns:</u>

  the X,Y co-ordinates of new points in the ground coordinate system

- <u>Given:</u>

  The model co-ordinates x,y of points in the relatively oriented and levelled individual models.

Planimetric Block adjustment by independent Models



Planimetric Block adjustment by independent Models

- The model co-ordinates are in separate independent, local co-ordinate systems for each model.
- Each of this co-ordinate systems is displaced(*two translations Xu,Yu*), and rotated (*rotation angle K*) relative to the ground co-ordinate system and has an arbitrary scale (*scale factor m*).

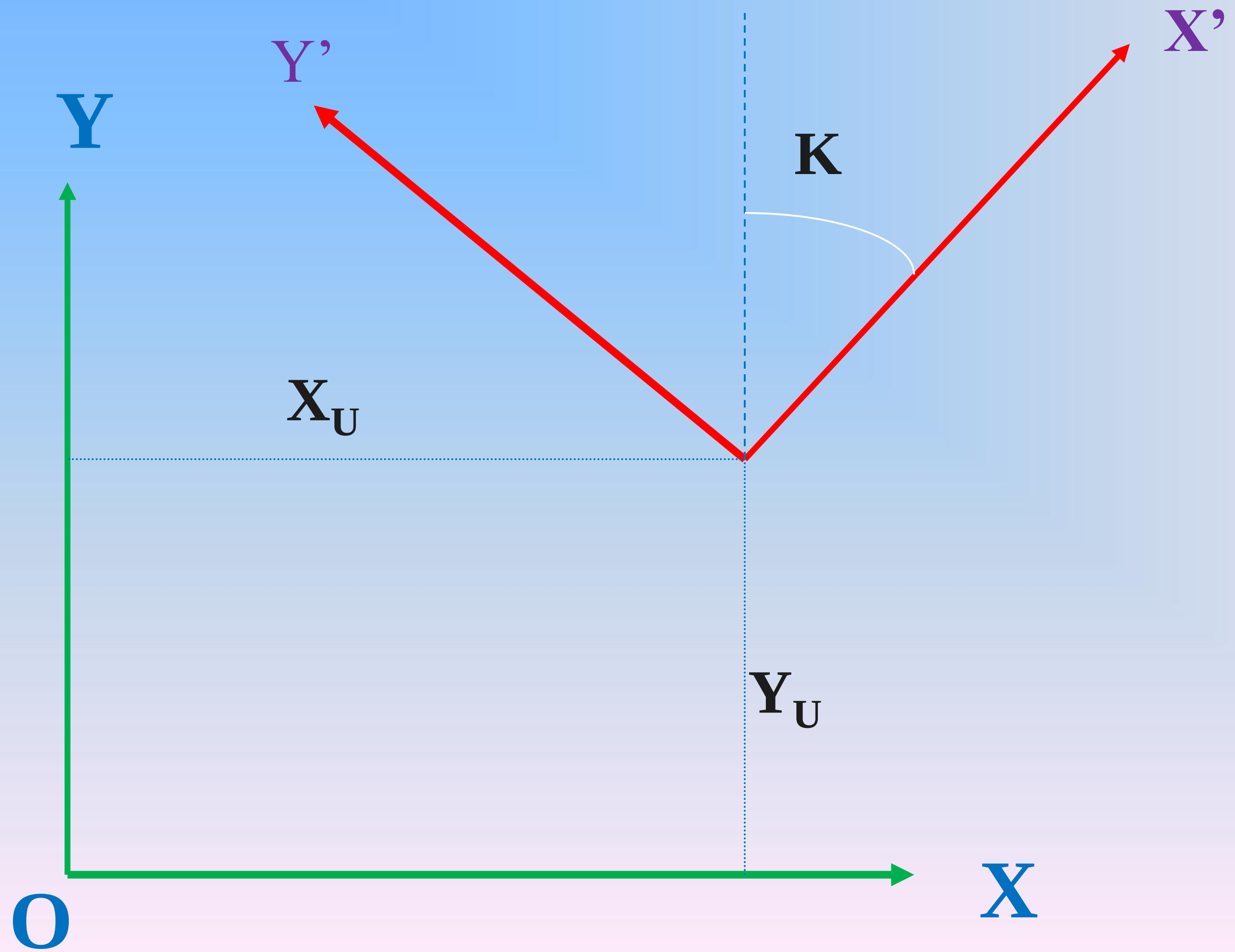
- An initial data to bring the individual models into one block in the co-ordinate system we have the tie points (pass points being 5, 6, 7, 8 and 9), the model and ground co-ordinates of the field surveyed control points.

THE ‘Adjustment’

- The models are:
  - Displaced (two translations Xu, Yu)
  - Rotated (rotation angle K)
  - Scaled (scale factor m)

- So that;
- The tie points fit together as well as possible and
- The residual discrepancies at the control points are small as possible.

The models



Mathematical Formulation

- The mathematical relation between a model and the ground coordinate system can be formulated as follows (plane similarity transformation)

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} X_U \\ Y_U \end{pmatrix} + m \begin{pmatrix} \cos K & -\sin K \\ \sin K & \cos K \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- The m and K values are the unknowns and the non linearity's can be eliminated by substituting

Mathematical Formulation

$$m * \cos K = a$$

$$m * \sin K = b$$

- With this substitutions the linear equations below are obtained.

$$X = X_U + a * x - b * y$$

$$Y = Y_U + a * y + b * x$$

Mathematical Formulation

- The extension of this equation system to a block of photographs is called a chained plane similarity transformation.

- The observation equations for the control point:

$$v_X = \overline{X}_U + \overline{a} * x - \overline{b} * y - X$$

$$v_Y = \overline{Y}_U + \overline{a} * y + \overline{b} * x - Y$$

Mathematical Formulation

- Observation equations for a tie point:

$$v_X = \overline{X}_U + \overline{a} * x - \overline{b} * y - \overline{X} - 0$$
$$v_Y = \overline{Y}_U + \overline{a} * y + \overline{b} * x - \overline{Y} - 0$$

- The overlined terms in the equations are the unknowns. The observation equations have an unusual form: vx and vy are to be interpreted as corrections to the (known) ground co-ordinates X,Y and to the imaginary observations “0”.

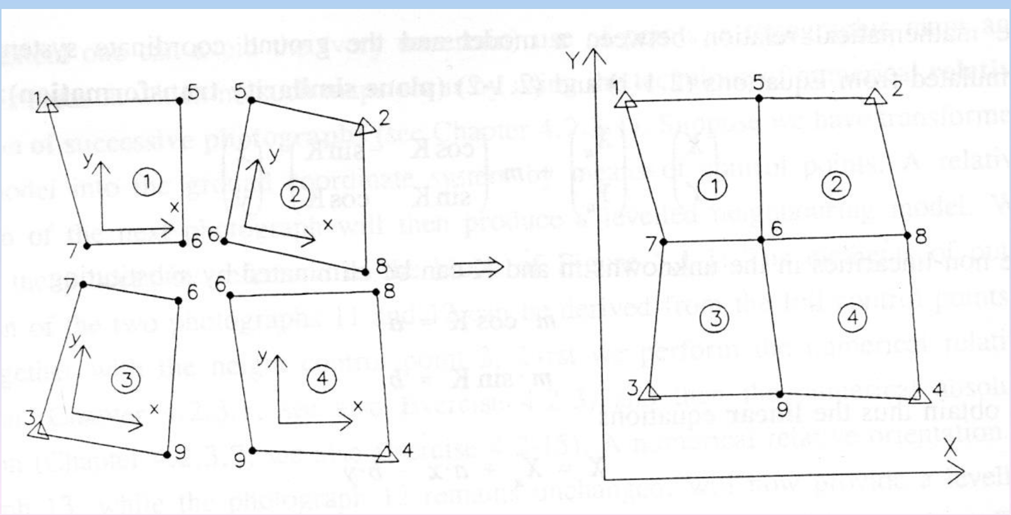
Mathematical Formulation

Unknowns: 4x4 = 16 Transformation Elements X_u,Y_u, a,b

5x2 = 10 tie point co-ordinates X,Y

total = 26

Observations: 32 model co-ordinates x,y



Observations

Control Points (1,2,3,4)

4x2=8

Tie Points

5 → 2x2=4

6 → 4x2=8

7 → 2x2=4

8 → 2x2=4

9 → 2x2=4 Total=32

Observation Equations

	X_u^1	Y_u^1	X_u^2	Y_u^2	X_u^3	Y_u^3	X_u^4	Y_u^4	a^1	b^1	a^2	b^2	a^3	b^3	a^4	b^4	X_5	Y_5	X_6	Y_6	X_7	Y_7	X_8	Y_8	X_9	Y_9	1	
v_x	1								x_1^1	$-y_1^1$																	X_1	Model 1
v_y		1							y_1^1	x_1^1																	Y_1	
v_x	1								x_5^1	$-y_5^1$							-1											
v_y		1							y_5^1	x_5^1								-1										
v_x	1								x_6^1	$-y_6^1$									-1									
v_y		1							y_6^1	x_6^1										-1								
v_x	1								x_7^1	$-y_7^1$											-1							
v_y		1							y_7^1	x_7^1												-1						
v_x			1								x_5^2	$-y_5^2$					-1											Model 2
v_y				1							y_5^2	x_5^2						-1										
v_x			1								x_2^2	$-y_2^2$															X_2	
v_y				1							y_2^2	x_2^2															Y_2	
v_x			1								x_8^2	$-y_8^2$											-1					
v_y				1							y_8^2	x_8^2												-1				
v_x			1								x_6^2	$-y_6^2$							-1									
v_y				1							y_6^2	x_6^2								-1								
v_x					1								x_7^3	$-y_7^3$							-1							Model 3
v_y						1							y_7^3	x_7^3								-1						
v_x					1								x_6^3	$-y_6^3$					-1									
v_y						1							y_6^3	x_6^3						-1								
v_x					1								x_3^3	$-y_3^3$													X_3	
v_y						1							y_3^3	x_3^3													Y_3	
v_x					1								x_9^3	$-y_9^3$											-1			
v_y						1							y_9^3	x_9^3												-1		
v_x							1								x_6^4	$-y_6^4$			-1									Model 4
v_y								1							y_6^4	x_6^4				-1								
v_x							1								x_8^4	$-y_8^4$							-1					
v_y								1							y_8^4	x_8^4								-1				
v_x							1								x_9^4	$-y_9^4$									-1			
v_y								1							y_9^4	x_9^4										-1		
v_x							1								x_4^4	$-y_4^4$											X_4	
v_y								1							y_4^4	x_4^4											Y_4	

Upper index = model number

Lower index = point number

	X_u^1	Y_u^1	X_u^2	Y_u^2	X_u^3	Y_u^3	X_u^4	Y_u^4	a^1	b^1	a^2	b^2	a^3	b^3	a^4	b^4	X_5	Y_5	X_6	Y_6	X_7	Y_7	X_8	Y_8	X_9	Y_9	1	
v_x	1								x_1^1	$-y_1^1$																	X_1	Model 1
v_y		1							y_1^1	x_1^1																	Y_1	
v_x	1								x_5^1	$-y_5^1$							-1											
v_y		1							y_5^1	x_5^1								-1										
v_x	1								x_6^1	$-y_6^1$									-1									
v_y		1							y_6^1	x_6^1										-1								
v_x	1								x_7^1	$-y_7^1$											-1							
v_y		1							y_7^1	x_7^1												-1						

v_x	1		x_5^2	$-y_5^2$	-1							
v_y		1	y_5^2	x_5^2		-1						
v_x	1		x_2^2	$-y_2^2$							X_2	Model 2
v_y		1	y_2^2	x_2^2							Y_2	
v_x	1		x_8^2	$-y_8^2$					-1			
v_y		1	y_8^2	x_8^2						-1		
v_x	1		x_6^2	$-y_6^2$			-1					
v_y		1	y_6^2	x_6^2				-1				

v_x	1		x_7^3	$-y_7^3$			-1					
v_y		1	y_7^3	x_7^3				-1				
v_x	1		x_6^3	$-y_6^3$	-1							
v_y		1	y_6^3	x_6^3		-1						Model 3
v_x	1		x_3^3	$-y_3^3$							X_3	
v_y		1	y_3^3	x_3^3							Y_3	
v_x	1		x_9^3	$-y_9^3$					-1			
v_y		1	y_9^3	x_9^3						-1		

v_x	1		x_6^4	$-y_6^4$	-1							Model 4
v_y		1	y_6^4	x_6^4		-1						
v_x	1		x_8^4	$-y_8^4$			-1					
v_y		1	y_8^4	x_8^4				-1				
v_x	1		x_9^4	$-y_9^4$					-1			
v_y		1	y_9^4	x_9^4						-1		
v_x	1		x_4^4	$-y_4^4$							X_4	
v_y		1	y_4^4	x_4^4							Y_4	

Upper index = model number

Lower index = point number

Model 1			Model 2		
Pt.No.	x	y	Pt.No.	x	y
1	148.29	573.28	2	366.93	558.43
5	374.11	561.87	5	154.36	561.30
6	362.77	147.41	6	130.40	143.24
7	138.27	151.39	8	358.30	140.28

Model 3			Model 4		
Pt.No.	x	y	Pt.No.	x	y
3	148.59	139.40	4	359.38	135.30
6	362.10	542.71	6	140.31	578.42
7	159.40	556.85	8	359.34	549.19
9	345.67	128.76	9	141.97	149.87

Ground coordinates of control points in m		
Pt.No.	X	Y
1	4443.81	8338.54
2	7658.37	7993.67
3	4472.02	1071.18
4	8348.54	1316.60

The image-to-ground coordinate relationship is established through the collinearity model and is represented by the collinearity equations:

$$x = x_p - c\frac{r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)}{r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)}$$

$$y = y_p - c\frac{r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)}{r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)}$$

The above equations involve the following quantities:

- The measured image point coordinates: x, y
- Interior orientation parameters of the camera: x_p, y_p, c
- Exterior orientation parameters of the image under consideration: $X_o, Y_o, Z_o, \omega, \phi, \kappa$

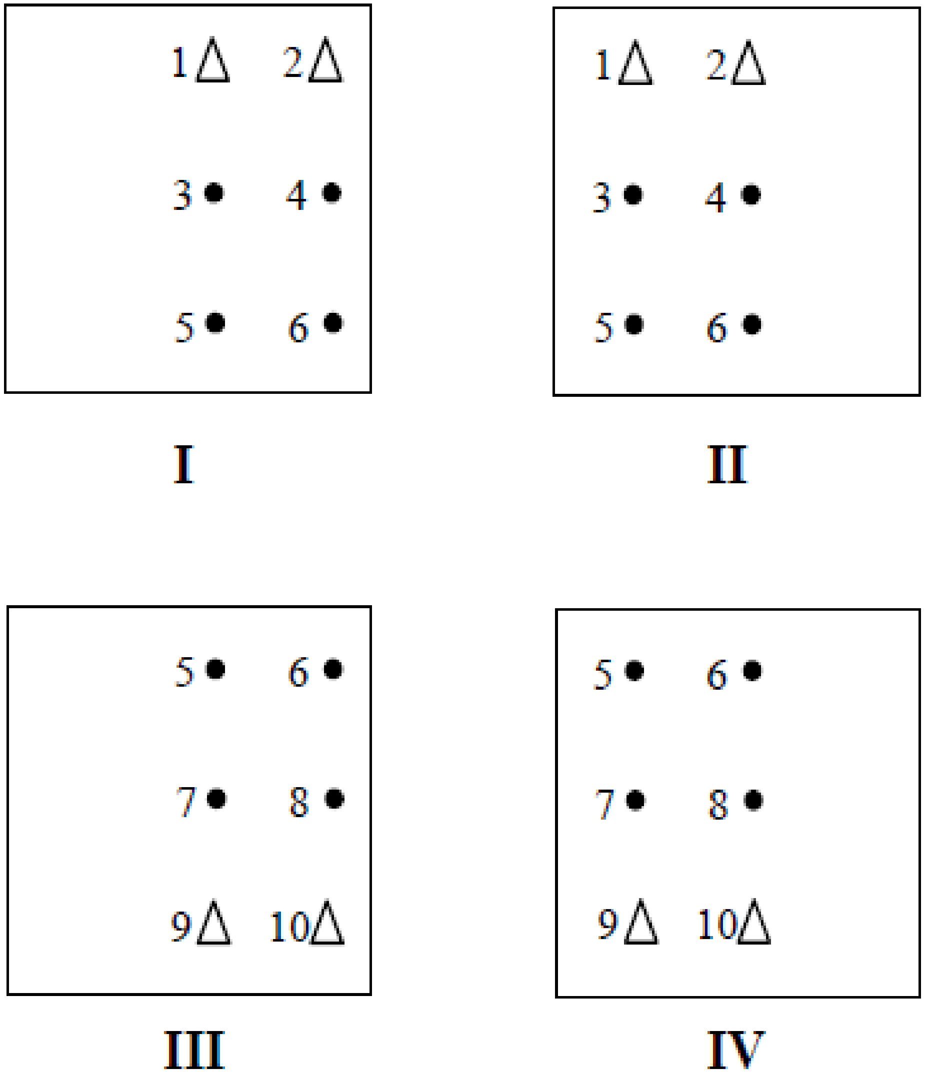
  where ω, ϕ, κ are embedded in the rotation matrix components:

$r_{11} = \cos\phi \cos\kappa$	$r_{12} = -\cos\phi \sin\kappa$	$r_{13} = \sin\phi$
$r_{21} = \cos\omega \sin\kappa + \sin\omega \sin\phi \cos\kappa$	$r_{22} = \cos\omega \cos\kappa - \sin\omega \sin\phi \sin\kappa$	$r_{23} = -\sin\omega \cos\phi$
$r_{31} = \sin\omega \sin\kappa - \cos\omega \sin\phi \cos\kappa$	$r_{32} = \sin\omega \cos\kappa + \cos\omega \sin\phi \sin\kappa$	$r_{33} = \cos\omega \cos\phi$

- The ground coordinates of point: X, Y, Z.

Given:

Four images in two strips



Points 1, 2, 9, 10 are Full Control Points

Points 3, 4, 5, 6, 7, 8 are tie points (i.e., they appear in more than one image and have unknown ground coordinates)

- **Required:**
- If we need to solve for the EOP's of the four images and the ground coordinates of tie points assuming known/errorless IOP's and ground coordinates of control points, analyze the observations-parameters and show the structure of each component of the *Observation Equation Model* (*Gauss-Markov model*) required to perform a least squares adjustment procedure;
- $y = Ax + e$.

- **Observation Equation Model:**

$$y = A\,x + e$$

where,

y: vector of observations, A: the design matrix, x : vector of parameters, e: vector of errors.

SOLUTION

- **Observation-parameters analysis:**
- a) The observations
- The measured image coordinates of points 1,2,3,4,5,6 on image I — 6 x 2 =12
- The measured image coordinates of points 1,2,3,4,5,6 on image II — 6 x 2 =12
- The measured image coordinates of points 5,6,7,8,9,10 on image III — 6 x 2 =12
- The measured image coordinates of points 5,6,7,8,9,10 on image IV — 6 x 2 =12
- **Total n = 48**

- **b) The parameters/unknowns**
- Known interior orientation parameters for the camera: xp, yp, c — 0 (erroless)
- Exterior orientation parameters:

  Xo, Yo, Zo, ω, φ, κ for images I,II,III,IV — 4 x 6 =24
- The ground coordinates of the control points 1,2,9,10 — 0 (erroless)
- The ground coordinates of the tie points 3,4,5,6,7,8 — 6 x 3 =18
- **Total m = 42**

- <u>c) Redundancy</u>
- n – m = 48 - 42 = 6.
- Due to this redundancy, the unknown parameters are determined through a least squares adjustment procedure.

Bundle Block Adjustment

- ONE PHOTOGRAPH IS TAKEN AS A UNIT !!!!

- **Definition :** Numerical orientation of the two bundles of rays of a stereopair of photographs.

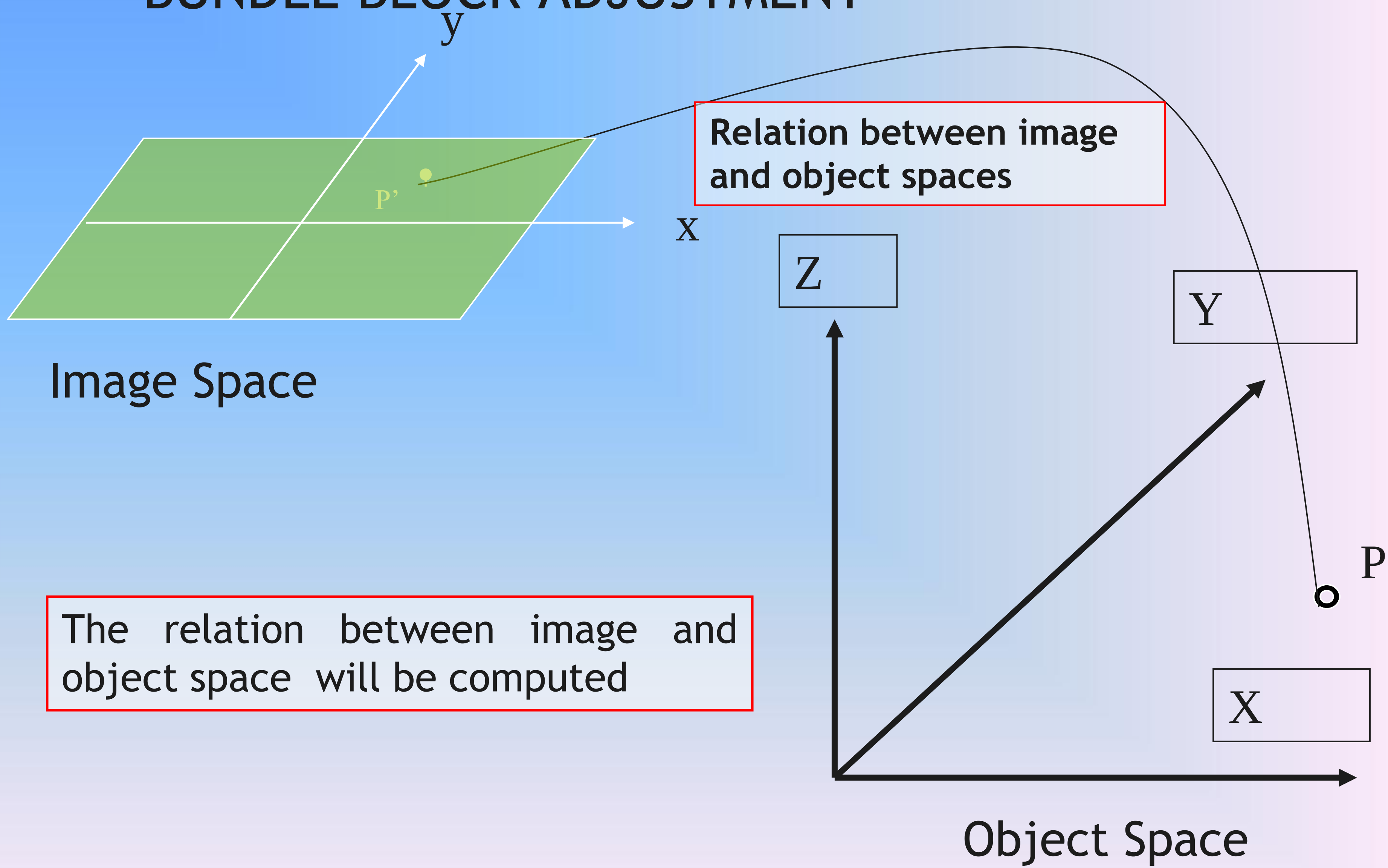
Bundle Block Adjustment

- KNOWN:
- Co-ordinates of control points

- UNKNOWN:
- 12 elements of outer orientation of two photographs
- XYZ of new points

- MEASURED:
- Image co-oordinates of all points

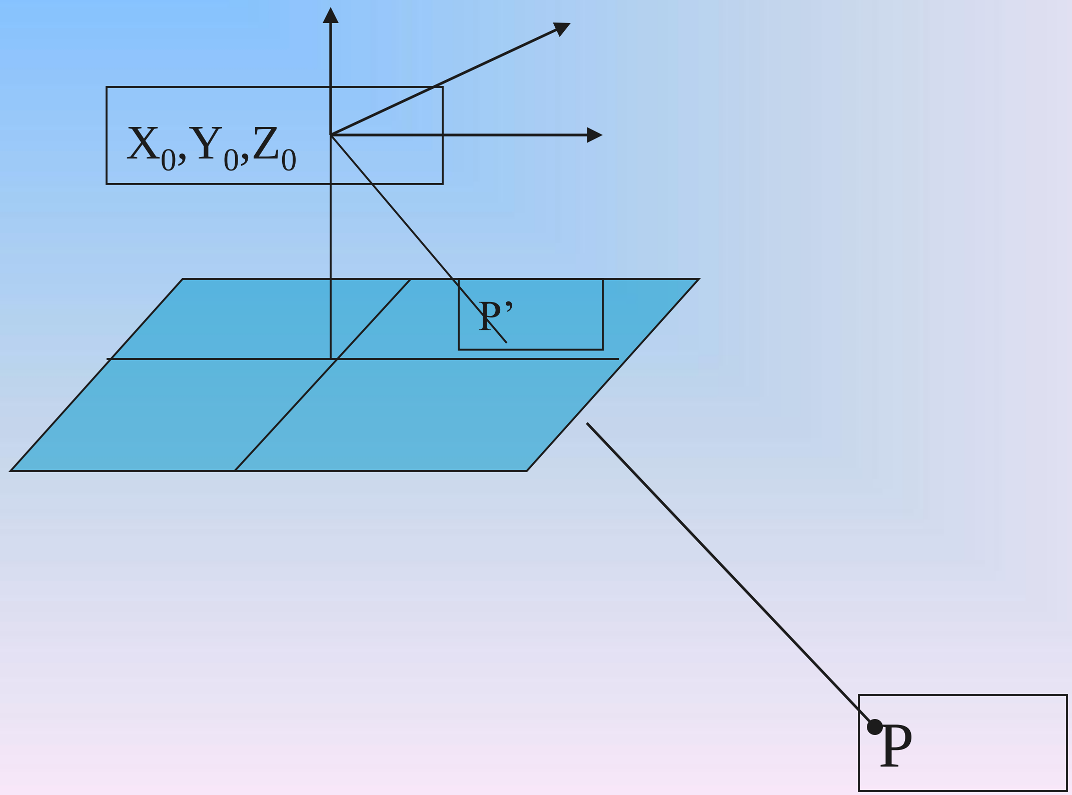
BUNDLE BLOCK ADJUSTMENT



The relation between image and object space will be computed

Some Definitions

- The image co-ordinates and the associated projection centre of a photograph define a spatial bundle of rays

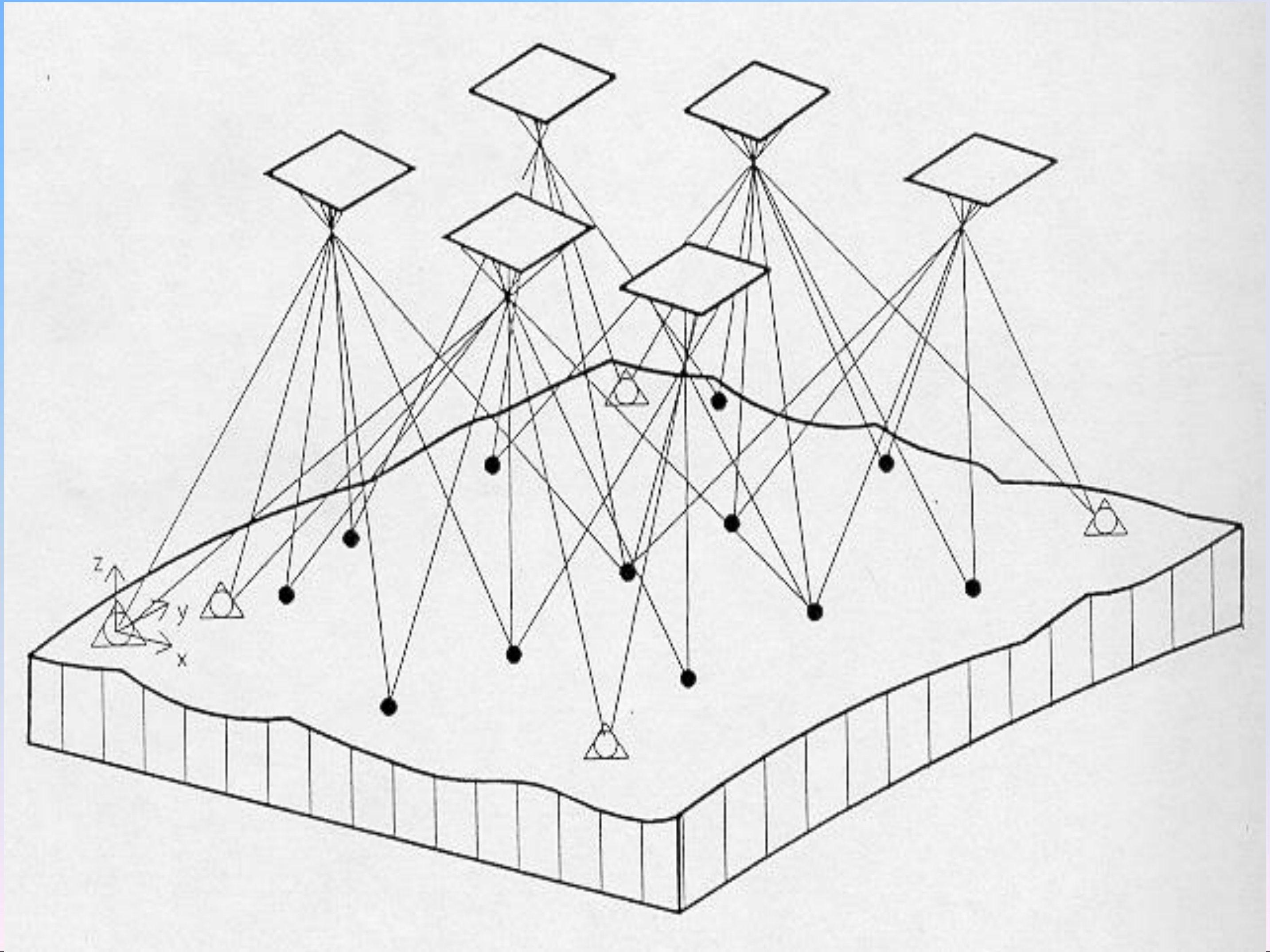


The Least Squares Adjustment Principle of the Bundle of Rays

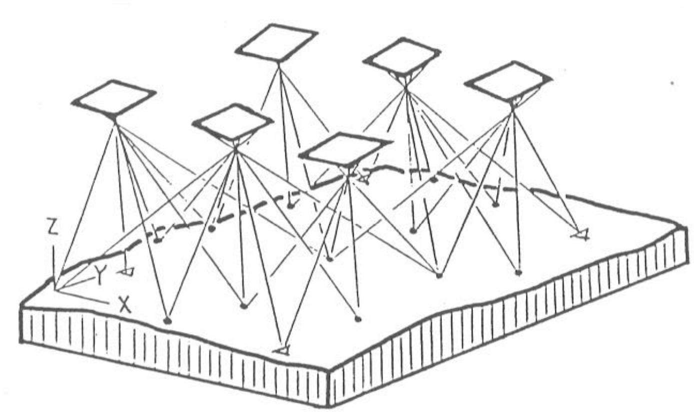
- The Bundles of rays are
- Displaced (three translations X0, Y0, Z0) and
- Rotated (three rotations ω, ϕ, κ)

- So that the bundles
- Intersect each other as well as possible at the tie points and
- Pass through the control points as nearly as possible

Principle



SUMMARY



- principle : simultaneous evaluation of image blocks (n>2)
- objective : control points / point determination (X,Y,Z)

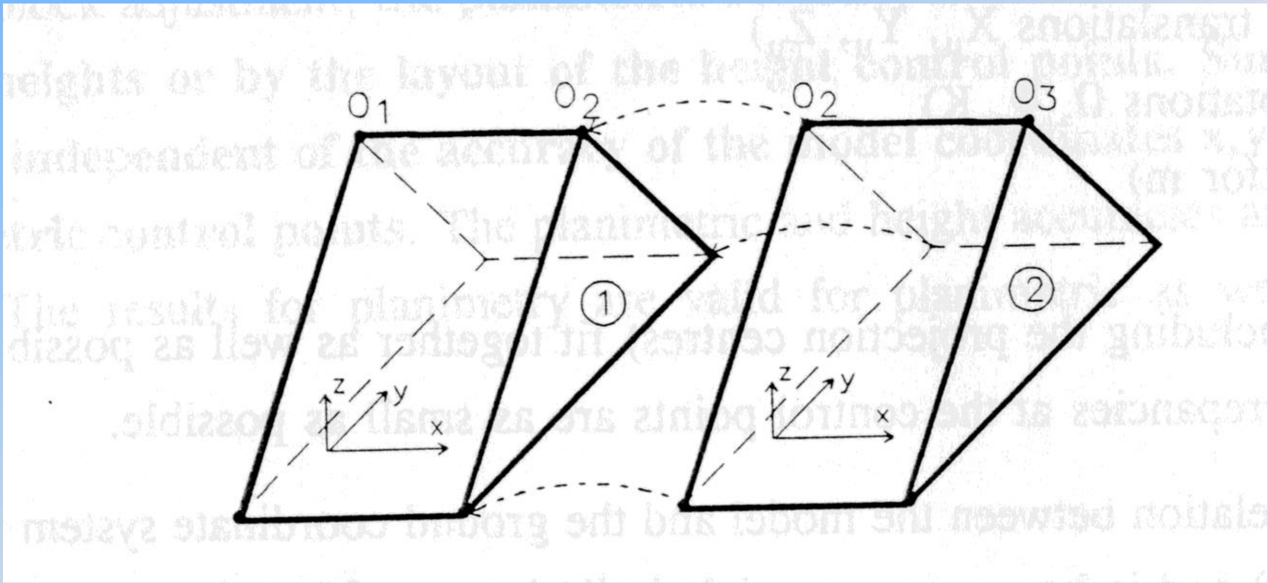
- bundle block adjustment
  - least-squares parameter estimation using image coordinates x' , y' of ground control points, tie points, and points to be determined
  - result : exterior orientation parameters + object space coordinates of points
- independent model adjustment
  - model coordinates instead of using image coordinates

Spatial Block Adjustment

- In spatial block adjustment we compute the XYZ co-ordinates of points in the ground co-ordinate system.
- As initial data we have the xyz model co-ordinates of points in the models formed by relative orientation and the model co-ordinates of the projection centers derived from the numerical relative orientation.

- The projection centers stabilize the heights along the strip. However, triangulation strips can not be stabilized at sides. Hence, elevation control point chains are required. A very good stabilization of heights in the block could only be achieved by sidelaps between strips of about 60%

Spatial Block Adjustment



MODEL CONNECTION WITH PROJECTION CENTERS

Spatial Block Adjustment