The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

MAT 4119 - Engineering Mathematics III

Tutorial Sheet 8

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1. Evaluate each of the following, giving your answer in the form $a+ib$, $a,b\in\mathbb{R}$. Where applicable, use the principal argument or principal branch of the logarithm.

   (a) $(-i)^i$ **(b)** $(1+i\sqrt{3})^{3+i}$ **(c)** $\log(1-i)^{11}$ **(d)** $Log\left(\frac{1}{(1+i)^4}\right)$

   (e) $\dfrac{e^{\ln 3+\frac{201}{2}\pi i}}{3}$ **(f)** $\cos(\frac{3\pi}{2}+i)$ **(g)** $\tan(\frac{\pi}{6}-i)$ **(h)** $\sec(1-i)$

   (i) $\cosh^{-1}(i)$ **(j)** $(i+1)\sinh(2\pi i)$

2. Find the complex number(s), z such that

   (a) $z^3=-1+i$ **(b)** $\overline{iz+2i}=4$ **(c)** $z^4+6iz^2+16=0$

   (d) $e^z=1+i$ **(e)** $Log\, z+Log(2z)=\frac{3\pi}{2}$

3. Find the image under the mapping $f(z)=\sin z$ of the semi-infinite strip $S=\{z=x+iy : -\frac{\pi}{2}\leq x\leq\frac{\pi}{2}, y\geq 0\}$.

4. Evaluate the following limits:

   (a) $\lim\limits_{z\to 0}\left(\frac{Re\, z}{z}\right)$ **(b)** $\lim\limits_{z\to -\pi i} e^{\frac{z^2+\pi^2}{z+\pi i}}$

   (c) $\lim\limits_{z\to 2+i}(z+\frac{1}{z})$ **(d)** $\lim\limits_{z\to 0}(\sin\bar{z})$

   (e) $\lim\limits_{z\to\infty}\left(\frac{z^3+i}{z^3-i}\right)^2$ **(f)** $\lim\limits_{z\to\infty}\left(\frac{z^2+i}{z^3+3z^2+z+1}\right)$

5. Determine whether the following functions are continuous at the indicated point(s):

   (a) $f(z)=\frac{z-i}{z-1+3i}$ on $\mathbb{C}$

   (b) $f(z)=\begin{cases}\frac{z-i}{z^2+1} & z\neq i\\ -\frac{i}{2} & z=i\end{cases}$ at $z=i$

6. Determine the set where the following functions are analytic and find their derivative:

   (a) $f(x+iy) = x^3 - 3xy^2 + i(3x^2y - y^3)$

   (b) $f(z) = (1+z^2)^{\frac{3}{2}}$

   (c) $f(x+iy) = e^{-y}\cos x + ie^{-y}\sin x$

   (d) $f(z) = \dfrac{2z - i}{z + 2i}$

7. Evaluate $\int_C (2z+1)\,dz$, where C is

   (i) the simple line segment from $z = -i$ to $z = 0$

   (ii) two simple line segments c_1 and c_2 from $z = -i$ to $z = 0$ and from $z = 0$ to $z = 1$, respectively

   (iii) the circular arc $z = e^{i\theta}, \quad -\frac{\pi}{2} \leq \theta \leq 0$.

8. Evaluate $\int_C e^z\,dz$ along the line $y = x$ from $(-1,-1)$ to $(1,1)$.

9. Evaluate each of the following:

   (a) $\displaystyle\int_0^{\pi i} z\sin(z^2)\,dz$ **(b)** $\displaystyle\int_{-i}^{2i} (z+1)\,dz$ **(c)** $\displaystyle\int_0^{\pi} \sin^2 z\,dz$

10. Evaluate the following around a given positively oriented contour C:

    (a) $\displaystyle\oint_C \frac{1}{z^2 - 3z + 2}\,dz \qquad C : \left\{ z \in \mathbb{C} : |z - 1 - i| = \frac{1}{2} \right\}$

    (b) $\displaystyle\oint_C \frac{1}{z^3 + 4z}\,dz \qquad C : \{z \in \mathbb{C} : |z| = 1\}$

    (c) $\displaystyle\oint_C \frac{\sin^6 z}{z - \frac{\pi}{6}}\,dz \qquad C : \{z \in \mathbb{C} : |z| = 1\}$

    (d) $\displaystyle\oint_C \frac{e^{z^2}}{z^3}\,dz \qquad C : \{z \in \mathbb{C} : |z| = 5\}$