MEC 3102 – PRODUCTION ENGINEERING I AND ELECTRICITY & ELECTRONICS II

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3.5 Generation of Single-Phase

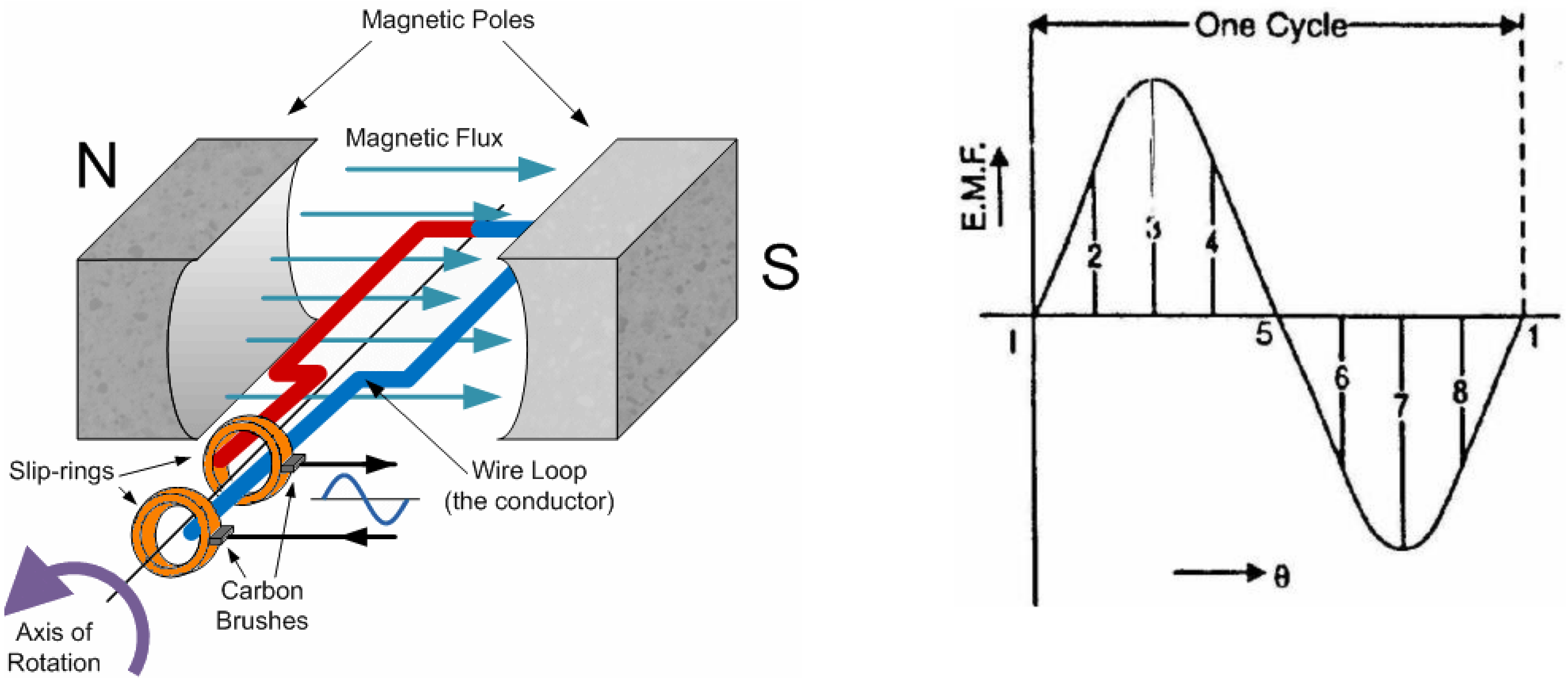


Fig. 3.8: Generated single phase e.m.f.

- The earliest application of alternating current was for heating the filaments of electric lamps. For this purpose the single-phase system was perfectly satisfactory.
- Some years later, a.c. motors were developed, and it was found that for this application the single-phase system was not very satisfactory. For instance, the single-phase induction motor – the type most commonly employed – was not self-starting unless it was fitted with an auxiliary winding.
- By using two separate windings with currents differing in phase by a quarter of a cycle or three windings with currents differing in phase by a third of a cycle, it was found that the induction motor was self-starting and had better efficiency and power factor than the corresponding single-phase machine.
- The system utilizing two windings is referred to as a *two-phase system* and that utilizing three windings is referred to as a *three-phase system*

3.6 Generation of three-phase e.m.f.s

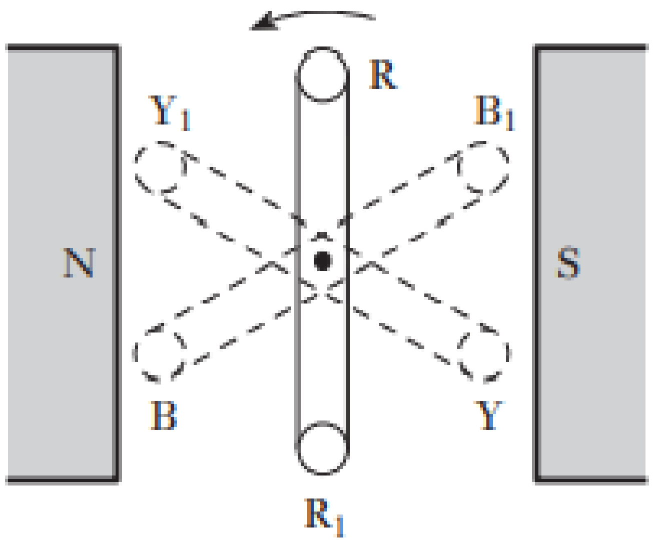


Fig. 3.9: Generation of three-phase e.m.f.s

- The letters R, Y and B are abbreviations of ‘red’, ‘yellow’ and ‘blue’, namely the colours used to identify the three phases.
- Also, ‘red–yellow–blue’ is the sequence that is universally adopted to denote that the e.m.f. in the yellow phase lags that in the red phase by a third of a cycle, and the e.m.f. in the blue phase lags that in the yellow phase by another third of a cycle.
- We shall refer to the slip-rings connected to sides R, Y and B as the ‘finishes’ of the respective phases and those connected to R1 , Y1 and B1 as the ‘starts’.

- Suppose the three coils are rotated anticlockwise at a uniform speed in the magnetic field due to poles N-S.
- The e.m.f. generated in loop RR_1 is zero for the position shown in Fig.3.9. When the loop has moved through 90° to the position shown in Fig.3.10., the generated e.m.f. is at its maximum value, its direction round the loop being from the 'start' slip-ring towards the 'finish' slip-ring.

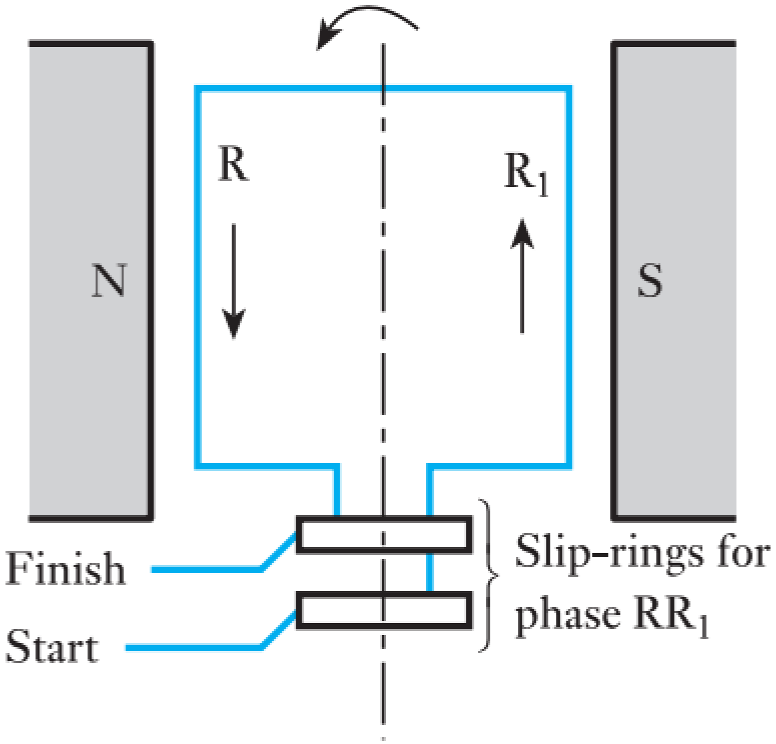


Fig.3.10: Loop $R\text{-}R_1$ at instant of maximum e.m.f.

- Let us regard this direction as positive; consequently the e.m.f. induced in loop RR_1 can be represented by the full-line curve of Fig.3.11.

- The e.m.f. generated in side Y of loop YY1 has exactly the same amplitude as that generated in side R, but lags by 120° (or one-third of a cycle)

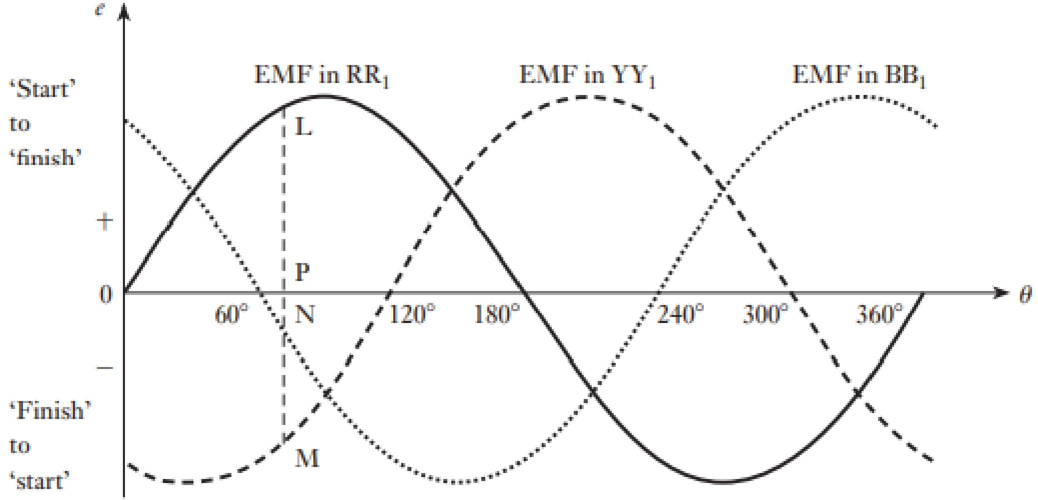


Fig.3.11: Waveforms of three-phase e.m.f.s

- Similarly, the e.m.f generated in side B of loop BB1 is equal to but lags that in side Y by 120°
- Hence the e.m.f.s generated in loops RR1 , YY1 and BB1 are represented by the three equally spaced curves of Fig.3.11, the e.m.fs being assumed positive when their directions round the loops are from 'start' to 'finish' of their respective loops.

If the instantaneous value of the e.m.f. generated in phase RR1 is represented by,

$$e = E_m \sin\theta,$$

then instantaneous e.m.f. in YY1 is

$$e = E_m \sin(\theta - 120°)$$

and instantaneous e.m.f. in BB1 is

$$e = E_m \sin(\theta - 240°)$$

Let the three phases of Fig.3.9, be represented as in Fig. 3.12

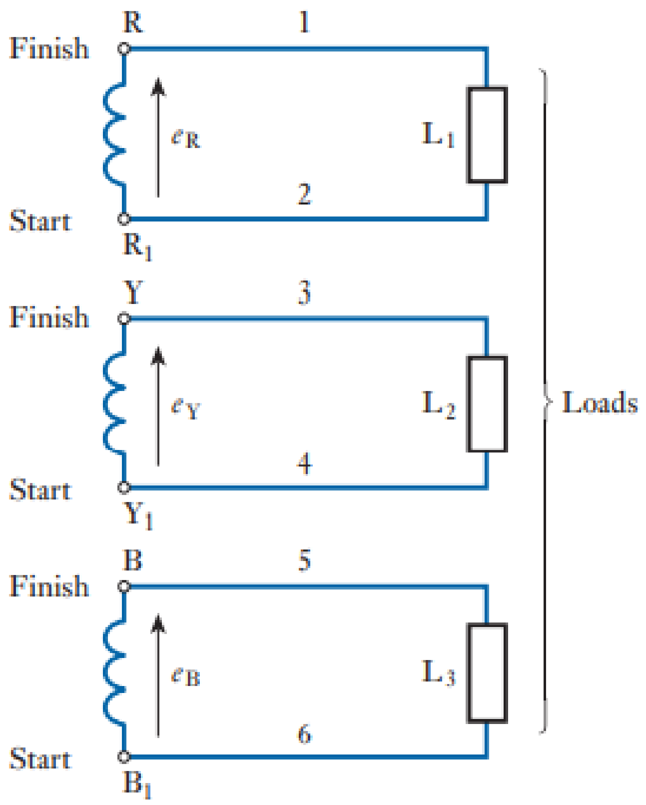


Fig.3.12: Three-phase windings with six line conductors

3.7 Delta connection of three-phase windings

- Let L1 , L2 and L3 represent loads connected across the respective phases in Fig.3.12.
- Let the e.m.f.s to be positive when acting from ‘start’ to ‘finish’ and be represented by the arrows e_R, e_Y, and e_B in Fig. 3.12.
- This arrangement necessitates six line conductors and is therefore cumbersome and expensive, so let us consider how it may be simplified.
- The ‘start’ of one phase should be connected to the ‘finish’ of another phase, so that the arrows representing the positive directions of the e.m.f.s point in the same direction round the mesh formed by the three windings.

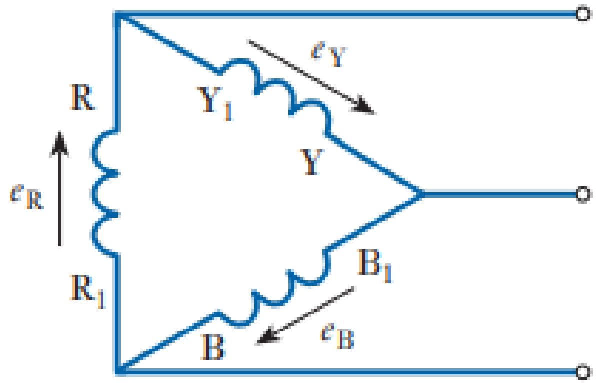
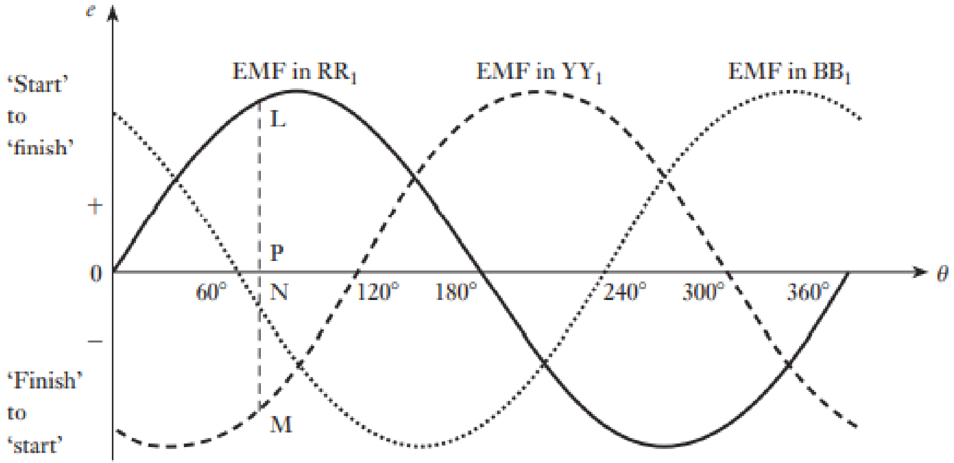


Fig.3.13: Conventional representation of a delta or mesh-connected winding

- At instant P in Fig.3.11, the e.m.f. generated in phase R is positive and is represented by PL acting from R1 to R. The e.m.f. in phase Y is negative and is represented by PM acting from Y to Y1 , and that in phase B is also negative and is represented by PN acting from B to B1.
- But the sum of PM and PN is exactly equal numerically to PL; consequently, the algebraic sum of the e.m.f.s round the closed circuit formed by the three windings is zero. Instantaneous value of the total e.m.f acting from B_1 to R is

$$e_R + e_Y + e_B$$
$$= E_m\{\sin\theta + \sin(\theta - 120°) + \sin(\theta - 240°)\}$$
$$= E_m(\sin\theta + \sin\theta \cdot \cos 120° - \cos\theta \cdot \sin 120° + \sin\theta . \cos 240° - \cos\theta \cdot \sin 240°)$$
$$= 0$$

Hence, no circulating current is present.



3.8 Star connection of three-phase windings

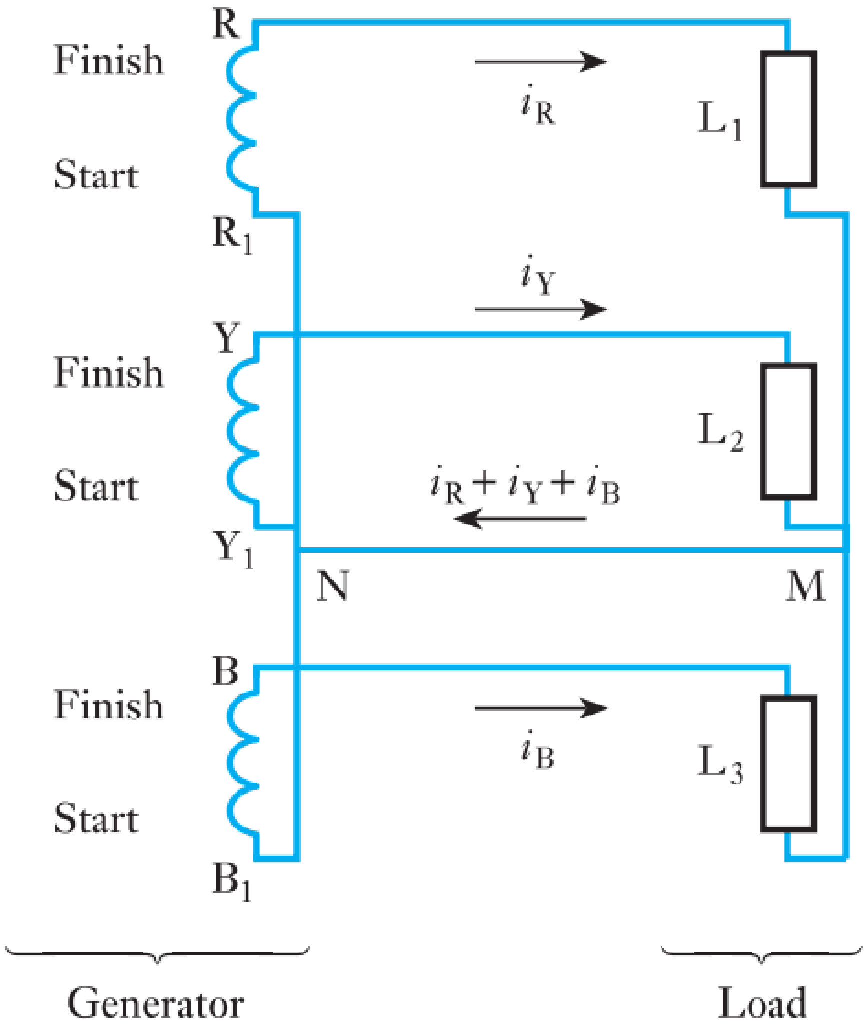


Fig.3.14: Star connection of three-phase winding

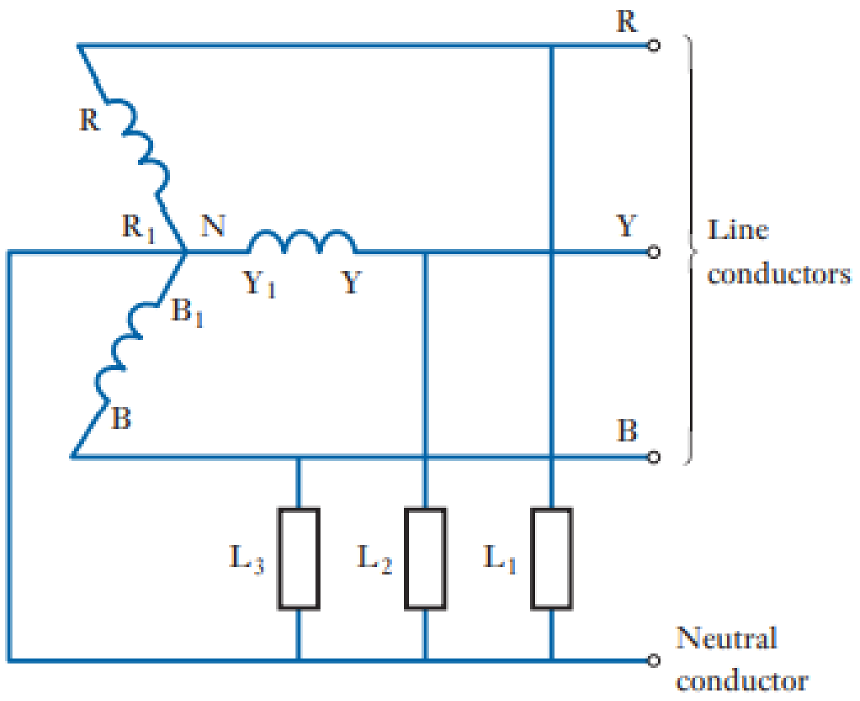


Fig.3.15: Four-wire star-connected system

- This arrangement is referred to as a *four-wire star-connected* system and is more conveniently represented as in figure 3.15, and junction N is referred to as the star or neutral point.
- Three-phase motors are connected to the line conductors R, Y and B, whereas lamps, heaters, etc. are usually connected between the line and neutral conductors, as indicated by L1, L2 and L3, the total load being distributed as equally as possible between the three lines.
- If these three loads are exactly alike, the phase currents have the same peak value, I_m, and differ in phase by 120°.

- Hence if the instantaneous value of the current in load L_1 is represented by,

$$i_1 = I_m \sin\theta$$

instantaneous current in L_2 is

$$i_2 = I_m \sin(\theta - 120^{\circ})$$

and instantaneous current in L_3 is

$$i_3 = I_m \sin(\theta - 240^{\circ})$$

- Hence instantaneous value of the resultant current in neutral conductor MN

$$\begin{aligned} i_1 + i_2 + i_3 \\ &= I_m\{\sin\theta + \sin(\theta - 120^{\circ}) \\ &+ \sin(\theta - 240^{\circ})\} \\ &= I_m \times 0 \\ &= 0 \end{aligned}$$

i.e. with a balanced load, the resultant current in the neutral conductor is zero at every instant

3.9 Voltages and currents in a Star-connected system

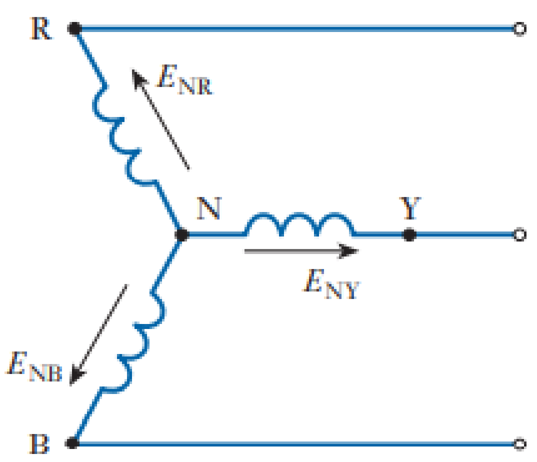


Fig.3.16: Star-connected generator

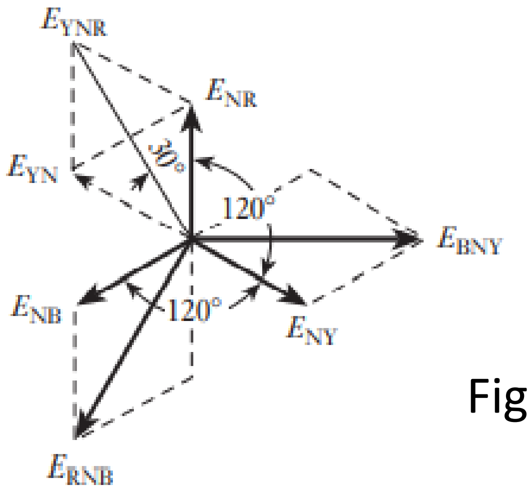


Fig.3.17: Phasor diagram

- The r.m.s. values of the e.m.f.s generated in the three phases can be represented by E_{NR}, E_{NY} and E_{NB}.

- The value of the e.m.f. acting from Y via N to R is the phasor difference of E_{NR} and E_{NB}.

Hence, E_{YN} is drawn equal and opposite to E_{NY} and added to E_{NR}, giving E_{YNR} as the e.m.f. acting from Y to R via N.

- From the symmetry of this diagram it is evident that the line voltages are equal and are spaced 120° apart.
- Since the angle between E_{NR} and E_{YN} is 60°,

$$E_{YNR} = 2E_{NR}\cos 30^{\circ} = \sqrt{3}E_{NR}$$

i.e.

Line voltage $= 1.73 \times$ star (or phase) voltage

- From Fig.3.16 it is obvious that in a star-connected system, the current in a line conductor is the same as that in the phase to which that line conductor is connected. Hence, in general, if

$V_L =$ p.d. between any two line conductors

$=$ line voltage

And

V_P = p.d. between a line conductor and the neutral point.

= star voltage (or voltage to neutral)

and if I_P and I_L are line and phase currents respectively, then for a star-connected system,

$$V_L = \sqrt{3} V_P \qquad (3.1)$$

and

$$I_L = I_P \qquad (3.2$$

Note: The voltage given for a three-phase system is always the *line voltage* unless it is stated otherwisee.

3.10 Voltages and currents in a delta-connected system

- Let I_1 , I_2 and I_3 be the r.m.s. values of the phase currents having their positive directions as indicated by the arrows in Fig.3.18.

- Since the load is assumed to be balanced, these currents are equal in magnitude and differ in phase by 120°.

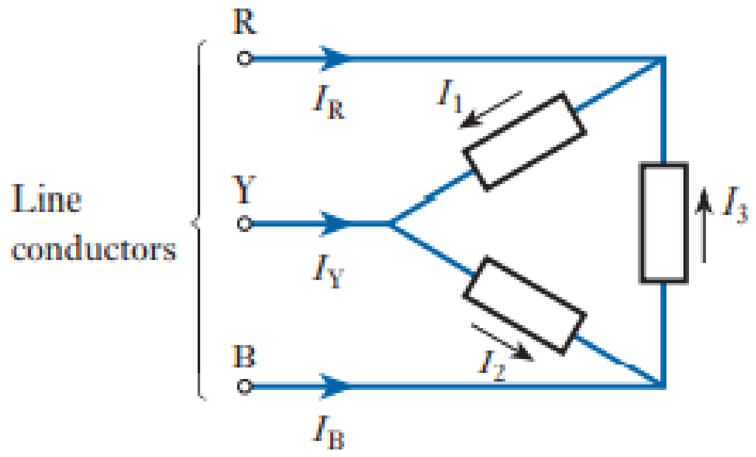


Fig.3.18: Delta-connected system with balanced load

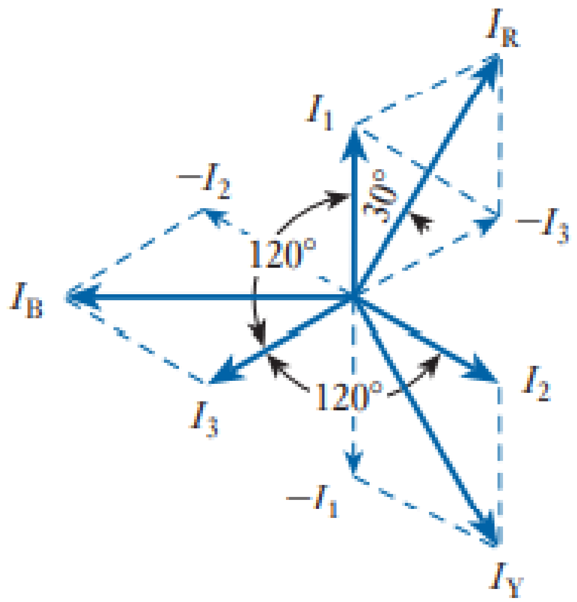


Fig.3.19: Phasor diagram

- From Fig. 3.18 it will be seen that I_1 , when positive, flows away from line conductor R, whereas I_3 , when positive, flows towards it. Consequently, I_R is obtained by subtracting I_3 from I_1 , as in Fig.3.19. Hence,

$$I_R = 2I_1 \cos 30^{\circ} = \sqrt{3}I_1$$

Hence, for a delta-connected system with a balanced load.

Line current $= \sqrt{3}$ phase current

$$I_L = \sqrt{3}I_P \tag{3.3}$$

and in a delta-connected system, the line and the phase voltages are the same, i.e.

$$V_L = V_P \tag{3.4}$$

3.11 Power in a three-phase system with a balanced load

- If I_P is the r.m.s. value of the current in each phase and V_P the r.m.s. value of the p.d. across each phase,

Active power per phase

$$= I_P V_P \times \text{power factor}$$

and

Total active power

$$= 3 I_P V_P \times \text{power factor}$$

$$P = 3 I_P V_P \cos \emptyset \qquad (3.5)$$

If I_L and V_L are the r.m.s. values of the line current and voltage respectively, then for a star-connected system,

$$V_P = \frac{V_L}{\sqrt{3}} \text{ and } I_P = I_L$$

Substituting for I_P and V_P in equation (3.5), we have

Total active power in watts

$$= \sqrt{3} I_L V_L \times \text{power factor}$$

For a delta-connected system

$$V_P = V_L \text{ and } I_P = \frac{I_L}{\sqrt{3}}$$

Again, substituting for I_P and V_P in equation (3.5), we have

Total active power in watts

$$= \sqrt{3} I_L V_L \times \text{ power factor}$$

Hence, it follows that, for any balanced load,

Active power in watts

$= \sqrt{3} \times$ line current $\times$ line voltage $\times$ power factor

$$\boldsymbol{P = \sqrt{3} I_L V_L \cos \emptyset} \qquad (3.6)$$

Example:

A three-phase motor operating off a 400 V system is developing 20 kW at an efficiency of 0.87 p.u. and a power factor of 0.82. Calculate:

(a) The line current;

(b) The phase current if the windings are delta-connected.

Solution:

(a) Since

$$\text{Efficiency} = \frac{\text{output power in watts}}{\text{input power in watts}}$$

$$\eta = \frac{\text{output power in watts}}{1.73 I_L V_L \times \text{p.f.}}$$

$$\therefore \qquad 0.87 = \frac{20 \times 1000}{1.73 \times I_L \times 400 \times 0.82}$$

and $\qquad \text{Line current} = I_L = \mathbf{40.0\ A}$

(b) For a delta-connected winding

$$I_P = \frac{I_L}{1.73}$$

$$I_P = \frac{I_L}{1.73} = \frac{40.0}{1.73} = \mathbf{23.1\ A}$$

3.12 Measurement of active power in a three-phase, three-wire system

a) Star-connected balanced load, with neutral point accessible

- If a wattmeter W is connected with its current coil in one line and the voltage circuit between that line and the neutral point, as shown in Fig.3.20, the reading on the wattmeter gives the power per phase:

Total active power

$= 3 \times$ wattmeter reading

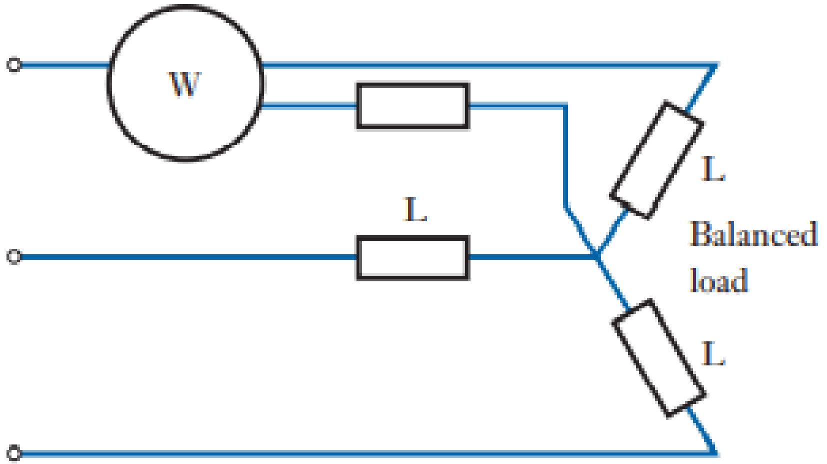


Fig.3.20: Measurement of active power in a star-connected balanced load

b) Balanced or unbalanced load, star- or delta-connected: The two-wattmeter method

Instantaneous power in load

$$L_1 = i_R V_{RN}$$
$$L_2 = i_R V_{RN}$$
$$L_3 = i_B V_{BN}$$

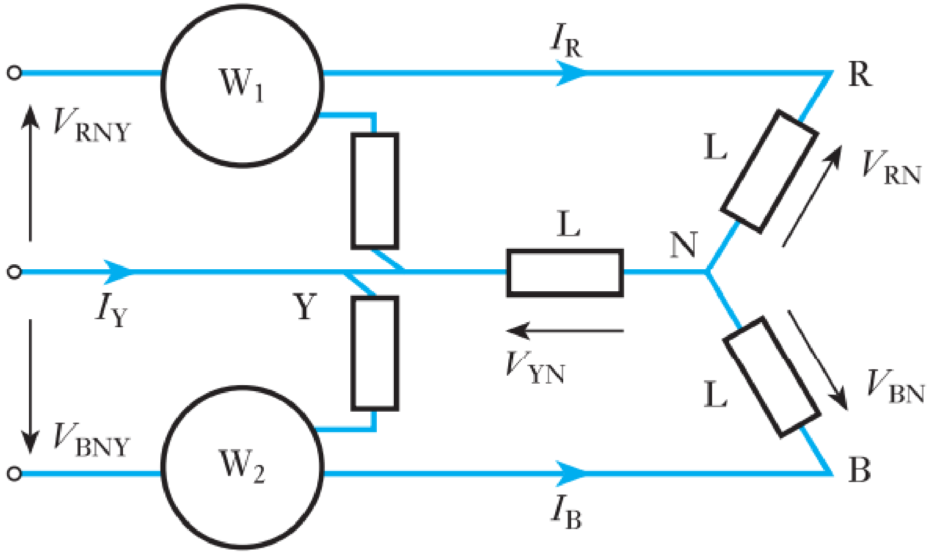


Fig.3.21: Measurement of power by two wattmeters

sum of instantaneous powers measured by W1 and W2 is,

Total instantaneous power

$$= i_R V_{RN} + i_Y V_{YN} + i_B V_{BN}$$

3.13 Power factor measurement by means of two wattmeters

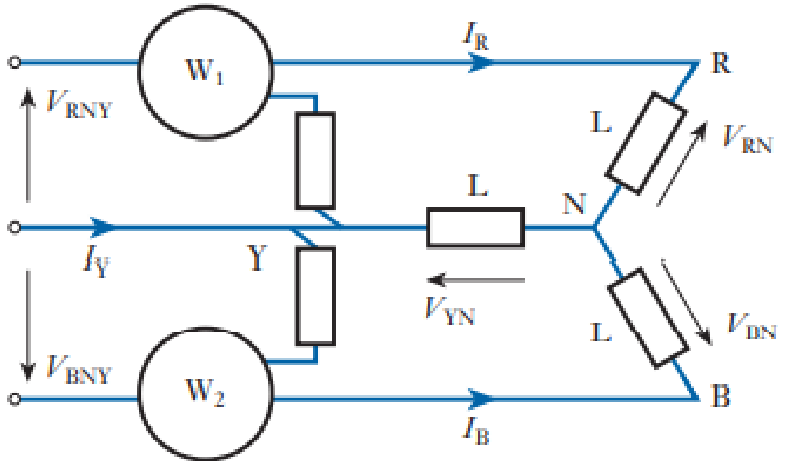
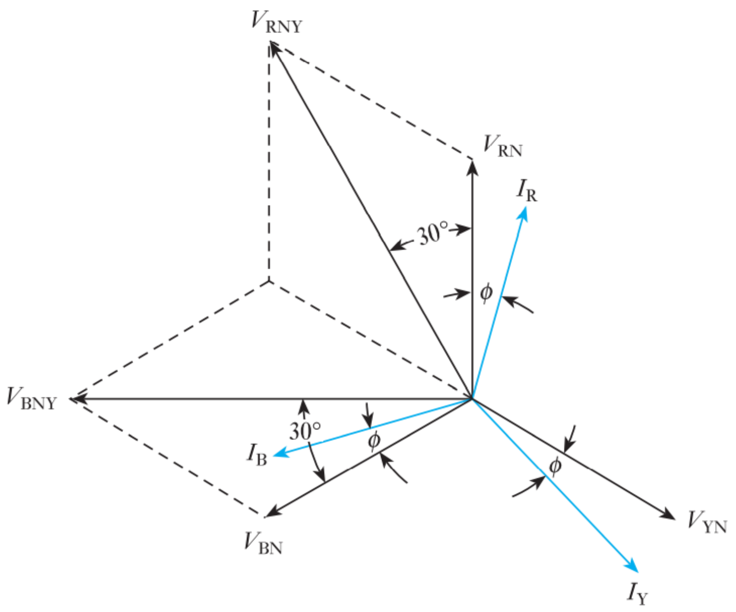


Fig.3.22: Measurement of active power and power factor by two wattmeters



- Let L represent three similar loads connected in star,

Phase difference between I_R and $V_{RNY} = 30^{\circ} + \emptyset$.

Therefore reading on W_1 is

$$P_1 = I_R V_{RNY} \cos(30^{\circ} + \emptyset)$$

Phase difference between I_B and $V_{BNY} = 30^{\circ} - \emptyset$.

Therefore reading on W_2 is

$$P_2 = I_B V_{BNY} \cos\left(30^{\circ} - \emptyset\right)$$

Since the load is balanced,

$$I_R = I_Y = I_B = (\text{say}) I_L, \text{numerically}$$

$$V_{RNY} = V_{BNY} = (\text{say}) V_L, \text{numerically}$$

Hence,

$$P_1 = I_L V_L \cos\left(30^{\circ} + \emptyset\right) \tag{3.7}$$

$$P_2 = I_L V_L \cos(30^{\circ} - \emptyset) \tag{3.8}$$

$$P_1 + P_2 = I_L V_L\{\cos(30^{\circ} + \phi) + \cos(30^{\circ} - \phi)\}$$

$$P_1 + P_2 = I_L V_L(\cos 30^{\circ} \cdot \cos \phi - \sin 30^{\circ} \cdot \sin \phi$$

$$+ \cos 30^{\circ} \cdot \cos \phi + \sin 30^{\circ} \cdot \sin \phi)$$

$$P_1 + P_2 = \sqrt{3} I_L V_L \cos \emptyset \tag{3.9}$$

This is an alternative method of proving that the sum of the two wattmeter readings gives the total active power, but it should be noted that this proof assumed a balanced load and sinusoidal voltages and currents.

Dividing equation (3.7) by equation (3.8), we have

$$\frac{P_1}{P_2} = \frac{\cos\left(30^{\circ} + \emptyset\right)}{\cos(30^{\circ} - \emptyset)} = (\text{say})y$$

$$y = \frac{\left(\sqrt{3/2}\right)\cos\emptyset - (1/2)\sin\emptyset}{\left(\sqrt{3/2}\right)\cos\emptyset + (1/2)\sin\emptyset}$$

so that

$$\sqrt{3}y\cos\emptyset + y\sin\emptyset = \sqrt{3}\cos\emptyset - \sin\emptyset$$

from which

$$\sqrt{3}(1-y)\cos\emptyset = (1+y)\sin\emptyset$$

Therefore,

$$3\left(\frac{1-y}{1+y}\right)^2 \cos^2\emptyset = \sin^2\emptyset = 1-\cos^2\emptyset$$

and

$$1 = \left\{1 + 3\left(\frac{1-y}{1+y}\right)^2\right\}\cos^2\emptyset$$

$$\text{Power factor} = \cos \emptyset = \frac{1}{\sqrt{\left\{1 + 3\left(\frac{1-y}{1+y}\right)^2\right\}}} \qquad (3.10)$$

An alternative method of deriving the power factor is as follows: From equations (3.7), (3.8) and (3.9)

$$P_1 - P_2 = I_L V_L \sin \emptyset$$

and

$$\tan \emptyset = \frac{\sin \emptyset}{\cos \emptyset} = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right) \qquad (3.11)$$

Example:

- The input power to a three-phase motor was measured by the two wattmeter method. The readings were 5.2 kW and −1.7 kW, and the line voltage was 400 V. Calculate:

(a) The total active power;

(b) The power factor;

(c) The line current.

Solution:

(a) Total power = 5.2 – 1.7 = **3.5 kW**.

(b) From equation [3.11],

$$\tan \phi = 1.73 \left\{ \frac{5.2 - (-1.7)}{5.2 + (-1.7)} \right\} = 3.41$$

$$\phi = 73°39'$$

and Power factor = $\cos \phi = \mathbf{0.281}$

From the data it is impossible to state whether the power factor is lagging or leading.

(c) From equation [3.6],

$$3500 = 1.73 \times I_L \times 400 \times 0.281$$

$$I_L = \mathbf{18.0\ A}$$

END OF LECTURE!